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POLARIZATION UTILIZATION IN RADAR TARGET RECONSTRUCTION

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The findings in this report are not to be construed as an official Department of the Navy position, unless so designated by other authorized documents.
This report is being dedicated to the late Dr. Herbert Mueller

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Radar Polarizaton, Scattering matrices (Jones, Sinclair, Stokes, Mueller, $[S]$, $[M]$, $[M_m]$); Optimal polarizations (Co-polarization and cross-polarization nulls). Verification of the target polarization null concept. Matrix transformation algorithms for inversion from $[S]$, $[M]$, $[M_m]$ and/or optimal polarization coordinates, and vice versa.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Basic theories for polarization utilization in radar target reconstruction are presented and a general literature review is given. It is shown that a scattering phenomenon can be uniquely expressed given the elements of either one of $[S]$, $[M]$, $[M_m]$ or the coordinates of the optimal polarization, i.e. unique inversion relations among the four equivalent representations exist which is relevant to target polarization synthesis. The developed theories are verified by computer computation using measurement and/or model scattering data as inputs.

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ABSTRACT

Basic theories for polarization utilization in radar target reconstruction are presented and a general literature review with many pertinent references is given.

The mathematical descriptions of polarization in terms of the Poincare polarization sphere are introduced and the relationships existing among the radar scattering matrix [S], the Stokes reflection matrix [M], the modified Mueller matrix [Mm], and the coordinates of the related co-polarization and cross-polarization nulls on the Poincare sphere are derived.

It is shown that a scattering phenomenon can be uniquely expressed given the elements of either one of [S], [M], [Mm] or the coordinates of the optimal polarizations, i.e. unique inversion relations among the four equivalent representations exist which is relevant to target polarization synthesis.

The developed theories are verified by computer computation using measurement data and/or model scattering data as inputs. The computer programs are listed and examples of our optimal polarization analysis are presented for the monostatic, relative phase case. Single perfectly conducting target shapes and some sea clutter testing models with and without target were chosen; and our studies demonstrate clearly that the optimal polarization concept introduced by KENNAUGH is very useful in radar target analysis as will be further pursued in other forthcoming reports.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

Unless otherwise stated, the symbols most commonly used in this report have the following meaning :

Greek Symbols

$\alpha_1, \alpha_2, \beta_1, \beta_2$	the elements of the unitary matrix [T]
α	the absolute phase of the antenna
$\gamma = \arctan\{\lvert P \rvert\}$	represents the magnitude of the polarization ratio
δ	the phase difference between y and x channels of the antenna
δ_x, δ_y	the phases of the x and y components of the polarization vector
$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$	Kronecker delta function
θ, ϕ'	colatitude and longitude on the Poincare sphere
θ_i, ϕ_i	represents the direction of the incident wave
θ_s, ϕ_s	represents the direction of the scattered wave
λ	free-space wavelength
ρ	complex transformation parameter
ρ^{co}, ρ^{x}	the COPOL and XPOL null roots
σ	radar cross section (RCS)
τ	the polarization ellipticity angle

ϕ	the polarization orientation angle
$\phi_{AA},\phi_{AB},\phi_{BA},\phi_{BB}$	the phase of the scattering matrix
ω	angular frequency (radians/sec.)

Latin Symbols

a	the magnitude of the polarization vector
a_x,a_y	the magnitudes of the (x,y) compoments of the polarization vector
AR	the axial ratio
$C_{\ell\ell},C_{\ell r},C_{rr}$	the circular compoments of the scattering matrix
e	base of natural logarithm
$\underline{E}$	electric field vector
E_x,E_y	the x and y components of the electric field vector
$\underline{E}^i,\underline{E}^s$	the incident and the scattered electric field vectors
$\underline{g}=(g_0,g_1,g_2,g_3)$ $=(I,Q,U,V)$	the Stokes vector
$\underline{g}_m=(g_{m0},g_{m1},g_{m2},g_{m3})$ $=(\frac{1}{2}(I+Q),\frac{1}{2}(I-Q),U,V)$	the Modified Stokes vector
$\underline{h}$	polarization vector
h	complex phasor of the polarization vector
$\hat{h}$	the unit vector in h direction

h_x, h_y	x and y phasor components of the polarization vector in x and y directions
$\hat{h}_x, \hat{h}_y$	unit vectors in x and y direction
h_ℓ, h_r	the left and right circular components of the polarization vector
h_A, h_B	the (A,B) components of the polarization vector
$\hat{h}_A, \hat{h}_B$	the unit vectors in (A,B) direction
h_A', h_B'	the (A',B') components of the polarization vector
$\underline{h}^i, \underline{h}^s$	the incident and the scattered polarization vectors
$\underline{H}$	magnetic field vector
$\underline{H}^i, \underline{H}^s$	the incident and scattered magnetic field vectors
j	$\sqrt{-1} = \exp\{j(\frac{1}{2}\pi)\}$
k	propagation constant
$\underline{k}$	propagation vector
$[M]$	the Mueller Matrix
m_{ij}	the Mueller matrix elements
$[Mm]$	the modified Mueller Matrix
M_{ij}	the modified Mueller matrix element
p	$=\mathrm{span}\{[S]\}$

P	polarization ratio
$\underline{r}$	position vector
R	radar range
[R]	constant transformation matrix relating [M] with [Mm]
[S]	the scattering matrix
$[S]_{SMA}$	the scattering matrix with absolute phase
$[S]_{SMR}$	the scattering matrix with relative phase
S_{xx}, S_{xy}, S_{yy}	the elements of the scattering matrix in (x,y) basis
$S_{AA}, S_{AB}, S_{BA}, S_{BB}$	the elements of the scattering matrix in (A,B) basis
$S'_{A'B'}, S'_{B'A'}, S'_{B'A'}, S'_{B'B'}$	the elements of the scattering matrix in (A',B') basis
[T]	unitary transformation matrix
u	auxiliary complex transformation paramater
x,y,z	cartesian coordinates

CHAPTER ONE

INTRODUCTION

In recent years there has been a rapidly expanding volume of research from both a theoretical and experimental point of view, directed towards the determination of the characteristic properties of radar targets through the use of polarization. The fact that makes this type of investigation possible is that the scattering properties of radar targets are dependent on the polarization of the incident radiation. This dependence which manifests itself as a depolarization of the scattered wave, is a function of the structure and geometry of the scatterer thus being characteristic for a particular target. It has been demonstrated that a radar target acts as a polarization transformer. This transformation was expressed (Sinclair 1948) as a matrix [S] which could be incorporated into the radar range equation. Kennaugh (1950) gave a geometrical meaning to the transformation by representing the power received by a radar on the Poincare sphere. In his series of reports (1949-1954), Kennaugh also demonstrated that there exist two radar polarization states for which the radar receives zero signal from the target, which are known as null polarizations. As it will be shown later the null polarizations (co-and cross-polarization nulls) along with the polarization orientation invariant, i.e. the span of the scattering matrix [S] ($\mathrm{span}\{[S]\}=\Sigma_i \Sigma_j |S_{ij}|^2$,where the S_{ij} are the elements of the matrix [S]), can be used to describe in their entirety the characteristic properties of a target at one frequency and for one aspect.

It has been established experimentally (Daley 1978-79, Weisbrod and Morgan 1979) that the null polarizations can be used in order to discriminate targets against scattering clutter, which is of great importance. This meets greatly the existing tremendous need for improved clutter rejection methods in order to detect accurately small surfaces which is also of great significance to military, geophysical and environmental remote sensing. It has been shown (Weisbrod et al, 1979) in the case of sea clutter that its non-random behaviour manifests itself as a characteristic clustering of co-and cross-polarized nulls as plotted on the Poincare sphere. This clustering was noticeably disturbed with the presence of a target. This phenomenon could lead to zero false alarm rate discriminants with the use of theoretical models extending existing clutter statistics, sensitive to the changes in the clustering of co-pol and cross-pol nulls.

In view of the fact that the co-and cross-polarization nulls along with the polarization transformation invariant (span of matrix [S]) are characteristic of a particular type of target for one aspect and at one frequency, these quantities should be given directly from the measurement data and recorded on the polarization sphere and/or its associated polarization maps.

So far, the polarization utilization in radar target related phenomena has been accomplished through the use of the radar scattering matrix [S] and the optimal polarization pairs of the co-and cross-polarization nulls have been expressed in term of the [S] matrix components. The use of the Stokes reflection matrix or Mueller matrix (Mueller, 1948) has been overlooked even though, it contains as much information as the scattering matrix and furthermore it is easier to obtain, since it involves only amplitude (power) measurements and not phase determination as in the case for the matrix [S]. There is also convincing evidence (Leader 1978, Boerner 1979) that the Mueller matrix elements behave in a manner characteristic of the material properties of the specific type of clutter they represent. Though the interpretation of this behaviour is in need of further investigation, one can safely assume that the information provided b the matrix [M] can be very useful in establishing target and particularly clutter characteristics especially for the incoherent scattering case. Due to the "additivity" property (Chandrasekhar 1950, Ishimaru 1978) of the Stokes parameters (Stokes, 1852) of independent waves, the independent incoherency properties of clutter are explicitly contained in the matrix [M] while they are only implicitly inherent in the [S] matrix. In addition, most of the experimental statistical clutter distributions are given , in terms of the components of [M]. Therefore, in order to interpret scatter characteristic polarization properties in terms of the associated co-and cross-polarization null statistics on the Poincare sphere, it would be highly practical to express these optimal polarizationa in terms of the Mueller matrix.

In view of the above, we have obtained here the scattering matrix elements from the Mueller matrix (both the matrix [M] and its modified form [Mm] for the bistatic and monostatic cases). Thus the problem of obtaining the scattering matrix with relative phases from the knowledge of the Mueller matrix is feasible. The expressions related to the problem were subsequently tested against real measurements for the case of sea clutter provided to us by Mr. J. Daley of the Naval Research Laboratory. In addition, we have shown that the elements of [S] ,[M] and/or [Mm] can be generated from the knowledge of the radius of the Poincaro sphere and from the spherical coordinates of either both co-polarization (COPOL) or one COPOL plus one cross-polarization(XPOL) null. This inverse relationship is important as it will assist us greatly in the target polarization synthesis problem.

The text of the present report is comprised of five Chapters. In Chapter Two a general review of the existing theoretical and experimental work in radar target polarization related phenomena is given. Chapter Three, covers the theoretical principles on which the Basic Polarization Descriptors, the Scattering matrix [S], its Transformation Invariants and Mueller Matrix [M] are based. In Section 3.6.2 of the same Chapter it is shown how the amplitudes and relative phases of the elements of the scattering matrix [S] are derived, given the Mueller matrix [M] or its modified form [Mm]. Also, in the same Chapter the relationship between [M] and [Mm] is given. Similarly, we show how the elements of [S] , [M] and/or [Mm] can be regenerated from the knowledge of the spherical coordinates of the two COPOL nulls or one COPOL plus one XPOL null. In Chapter Four the expressions obtained in Chapter Three, were tested satisfactorily against the data available to us. Also for the sake of completeness, we have calculated in Chapter Four the Mueller matrix elements (for both [M] and [Mm] matrices in the monostatic case) from the scattering matrix elements generated for various target shapes and sea clutter. In addition, the optimal polarization pairs of the co-and cross-polarized nulls were calculated and their coordinates on the Poincare sphere were found. Finally the summary of our results, conclusions and recommendations are given in Chapter Five. Proofs of some pertinent identities and computer programs for calculating relevant parameters are given in the appendices.

CHAPTER TWO

GENERAL REVIEW

The utilization of polarization in radar target and clutter studies has been the subject of rather extensive theoretical and experimental research efforts in recent years.

First Sinclair(1948) showed how a radar target can act as a polarization transformer and he was able to express this property with the use of a matrix which was incorporated in the radar range equation. There followed a series of papers by Booker, Rumsey, Deschamps, Kales and Bohnert(1951) on polarization, as it is related to radar antennas, which constituted the basis for future research on the subject. Huynen et al(1953) initiated from then on studies on radar returns by investigating the effects of polarization on radar scattering by ground targets and precipitation. Among the pioneers in the field of theoretical work on radar target scattering were Kennaugh(1949-1954) and Gent(1954). Kennaugh was also the first worker to introduce concepts of such potential impact as the optimal polarizations of a radar target(1949). Graves (1956) introduced the polarization power matrix which gives the total power back-scattered from the target for any transmitted polarization. Based on Kennaugh's work, Copeland(1960) gave a classification of the single radar target by measuring the received complex voltages using rotating linearly polarized antennas. Studies on polarization characteristics in the scattering from symmetric radar targets were reported by Crispin(1961), Bechtel and Ross(1962). and Huynen(1962, 1963). A summary of updating radar measurements was presented by Huynen, Landry, Webb and Allen at the Radar Reflectivity Measurements Symposium in 1964. In a Special IEEE Proceedings Issue on radar reflectivity (August 1965), the theory and measurement techniques for target scattering matrices (asymmetric objects included) were discussed by Lowenschuss(1965), Huynen(1965), Copeland(1960), Kuhl and Covelli(1965).

A vast amount of research has also been performed on time-varying distributed targets which is independent of the previously cited work on single radar targets. In his analysis of single targets, Gent(1954) considered also ensembles of distributions of single targets. Ament(1960) studied the problem of whether reciprocity holds for rough surface scattering and Ko(1962) gave an introduction with application to partially polarized scattering. Several statistical models for terrain are given by Spetner and Katz(1960). Studies of scattering from rough surfaces using scalar theory, which will remain classical, were treated by Beckmann and Spizzichino(1963) and updated by Beckmann(1965). Various rough surface models related to both theory and measurement have been studied by Beckmann(1965), Parks(1964) and Renau and Collinson(1965). Fung(March 1966, July 1966,1967) treated rough surface scattering using vector thoery which also considered depolarization of electromagnetic waves. Using high-frequency asymptotic theories based on the Kirchhoff approximation, Hagfors(1967) studied the depolarization of Lunar Radar Echoes and Stogryn(1967) worked on electromagnetic scattering from rough,finitely conducting surfaces. Van de

Hulst(1957) gave an introduction to high-frequency scattering. Worth mentioning here, are also the overall reviews of contributions to wave scattering from rough surfaces and various kinds of clutter, given in the Transactions of the IEEE Special Issue on Partial Coherence(1967), in "Radar Astronomy" edited by Evans and Hagfors(1968), in Bowman et al(1969), Crispin and Siegel(1968), in Ruck et al(1970) and in Beckmann's book on the depolarization of electromagnetic waves(1968). Huynen(1970) developed a phenomenological theory, applicable to all radar targets, according to which the radar target appears as an object for investigation through the process of the radar illumination. Thus one can avoid going into constructions of specific statistical and geometrical models, as it has been the case in most of the literature on the subject. The mathematical framework for the theory is given in terms of the target polarization scattering matrix, which has been used extensively in the Russian literature(Kanareykin et al, 1966, 1968; Stead, 1967; Zhivotovskiy, 1973; Potekhin et al, 1969; Kozolov, 1979, and Basalov et al, 1973).

Due to its importance, several efforts have been made recently to use the polarization sensitivity of radar targets towards a classification of radar targets. Thus the classification of radar targets suggested by Copeland has been recently investigated experimentally by Steinbach(1973, 1976). Studies on some methods for radar target identification are given by Von Schlachta(1977), Crom(1973), and Jeske(1976). There has also been an extensive amount of work to be found in the recent literature on precipitation scatter (Schneider and Williams, 1976; McCormick and Hendry 1976; Barge and Humphries, 1979, 1980; Hall et al, 1980; Bringi et al, 1979), on scattering from irregular surfaces(Bass and Fuks, 1975) and sea clutter(Valenzuela 1968, 1978). A detailed review on radar reflectivity of land and sea is given in the textbook of Long(1975) and also was discussed in Stiles and Holtzmann, 1979.

Of particular interest to the present work is the recently developed literature on studies of the co-and cross-polarization nulls of radar targets which promise to lead to very effective radar target versus clutter discrimination techniques. Recently, studies made in the Naval Research Laboratory(Daley 1978, 1979; Weisbrod and Morgan 1979) showed that, in the case of sea clutter the co-and cross-polarization nulls of clutter exhibit a non-random stable distribution when mapped on the Poincare sphere. This distribution was disturbed in the presence of a target, a phenomenon that may lead to effective target discrimination when fully investigated. The importance of the co-and cross-polarization nulls in radar target versus clutter discrimination has been emphasized particularly by Poelman(1971, 1974, 1976, 1977). Recently Ioannidis and Hammers(1979), Rosien et al(1979) have suggested schemes for radar identification in clutter, based on the characteristic properties of the co-polarization nulls. We also wish to refer to the forthcoming IEEE Trans. AP 29(2), March 1981, Special Issue on "Inverse Methods in Electromagnetics", in which several papers on polarization correction are presented.

Target versus clutter discrimination for targets embedded in terrain clutter has been improved by utilizing also depolarization effects(Gent et al, 1963; Weiss, 1967; Egan et al, 1967; Dainty, 1975). It was shown by Gough and Boerner(1979), Egan et al(1967) that the correlation functions of linearly and circularly polarized components allow discrimination between metal and dielectric random scatterers. In their studies Gough and Boerner(1979) are using the additive properties of the Mueller matrix in order to analyse the interaction of coherent target signal with various incoherent clutter components. The usefulness of the Mueller matrix(Mueller 1948, McCormick 1950) has also been recognized by Hagfors(1967), Leader(1978) and others.

Concluding we cite here other textbooks and papers related to radar target scattering as well as to the theory on partial coherence. Thus we distinguish here the works by Kraus(1966), Nathanson(1969), Thiel(1970), Meyer and Mayer(1973), Boerner (The State of the Art Review, 1978), Born and Wolf(1966), Ishimaru(1978), Strohbehn(1978); in the Russian literature the works by Kobzarev(1969), Shirman(1970), Gorshkov(1974), Kozlov(1979) and Zhivotovskiy(1976). We note here that in optics major contributions were made by R. Clark Jones(1941-1956) as was documented in Shurcliff(1962), Hecht and Zajac(1976) and Roots(1978).

CHAPTER THREE

THEORY

3.1 Introduction :

Chapter Three discusses in general the theoretical principles related to the utilization of polarization in radar target scattering phenomena. First, in Section 3.2, the basic descriptors of the polarization of monochromatic plane waves are given. In Sections 3.3 and 3.4, the theory of the radar cross-section scattering matrix is discussed and its transformation invariants as well as its representation on the Poincare sphere with the use of the co-and cross-polarization nulls are considered. In Section 3.5, the Stokes or Mueller matrix is reviewed and a method of reconstructing the scattering matrix, given its associated Mueller matrix is introduced in Section 3.6. In the same Section, the relationship between the Mueller and modified Mueller matrices is given. Finally in Section 3.7, the elements of the scattering and the Mueller matrices are regenerated from the knowledge of the spherical coordinates of either of the two co-polarization(COPOL) nulls or one COPOL plus one cross-polarization(XPOL) null.

3.2 Basic Polarization Descriptors :

3.2.1 The Polarization Vector

The polarization of an electromagnetic wave describes the orientation of the field vectors at a given point during one period of oscillation. In the present treatment, polarization is referred to the orientation of the electric field vector $\underline{E}$ only, since we are concerned with the far-field scattered radiation and the magnetic field vector $\underline{H}$ is perpendicular to $\underline{E}$ in direction and proportional to that of $\underline{E}$ in magnitude. The direction along which a wave propagates is given by the propagation vector $\underline{k}$. In an isotropic medium, the plane containing $\underline{E}$ and $\underline{H}$ and which is perpendicular to the direction of propagation, is called polarization plane [IEEE Standard 149, 1979]. In the case where the polarization of a wave is the same at every point in space, the wave is said to be linearly or plane polarized. We consider here monochromatic plane waves, i.e., plane waves at a single angular frequency ω, propagating along the z-axis of a rectangular coordinate system xyz, where the xy plane is a reference plane such as the mean surface of the earth. The electric field vector of a time-harmonic plane wave at a position $\underline{r}$ (x, y, z) and time t is given by

$$\underline{E}(\underline{r},t) = \underline{h}e^{j(\omega t-\underline{k}\cdot\underline{r})} = \underline{h}e^{j(\omega t-kz)} \tag{3.1}$$

where $\underline{k}$ is the propagation vector with magnitude $k = 2\pi/\lambda$, λ is the wavelength of the wave in free space, $\underline{h}$ is the complex electric field phasor, known as the complex polarization vector. In radar propagation the vector $\underline{h} = h\hat{h}$, with complex magnitude h and direction specified by the unit vector $\hat{h}$, may be decomposed along the two orthogonal directions x and y represented by the unit vectors $\hat{h}_x$ and $\hat{h}_y$, in the following manner :

$$\underline{h} = h\hat{n} = h_x\hat{h}_x + h_y\hat{h}_y \tag{3.2}$$

or

$$\underline{h} = \begin{bmatrix} h_x \\ \\ h_y \end{bmatrix} = \begin{bmatrix} a_x e^{j\delta_x} \\ \\ a_y e^{j\delta_y} \end{bmatrix} = \begin{bmatrix} a_x \\ \\ a_y e^{j\delta} \end{bmatrix} e^{j\delta_x} \qquad (3.3)$$

where a_x, a_y are the polarization vector component magnitudes, δ_x, δ_y are their phases and $\delta = \delta_y - \delta_x$ is their phase difference. In the antenna language, the magnitude of the polarization vector $|\, \underline{h} \,|^2 = a^2 = a_x^2 + a_y^2$ is a measure of antenna radiation efficiency and δ is the phase difference between the x and y channels of the antenna (Kraus, 1966).

3.2.2 The Polarization Ellipse

According to (3.1) and (3.3), the electric field vector $\underline{E}$ consists of two components along the x and y axes which are given by :

$$E_x = Re\{h_x e^{j(\omega t-kz)}\} = Re\{a_x e^{j\delta_x} e^{j(\qquad)}\}$$

$$= a_x \cos(\varepsilon + \delta_x) \qquad (3.4a)$$

$$E_y = Re\{h_y e^{j(\omega t-kz)}\} = Re\{a_y e^{j\delta_y} e^{j(\omega t-kz)}\}$$

$$= a_y \cos(\varepsilon + \delta_y) \qquad (3.4b)$$

or

$$\underline{E} = E_x \hat{h}_x + E_y \hat{h}_y = a_x \cos(\varepsilon+\delta_x)\hat{h}_x + a_y \cos(\varepsilon+\delta_y)\hat{h}_y \qquad (3.4c)$$

where $\varepsilon = \omega t - kz$, $Re\{\bullet\}$ stands for the real part of a complex number $\{\bullet\}$.

The curve that the electric field vector $\underline{E}$ describes at a typical point in space, is the locus of the points whose coordinates are given by (3.4). The nature of this curve is found after eliminating ε between Eqs.(3.4) and after some algebraic manipulation, to be given by :

$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2\left(\frac{E_x}{a_x}\right)\left(\frac{E_y}{a_y}\right)\cos\delta = \sin^2\delta \qquad (3.5)$$

which is the equation of an ellipse known as the polarization ellipse. The ellipse is inscribed into a rectangle whose sides are parallel to the coordinate axes and whose lengths are $2a_x$ and $2a_y$ (Fig. 3.1).

In Eq. (3.4) if $a_x = 0$, the wave is linearly polarized in the y-direction. If $a_y = 0$, the wave is linearly polarized in the x-direction. Following Kraus(1966), if $\delta = \delta_y - \delta_x = 0$ and $a_x = a_y$, the wave is also linearly polarized but in a plane at angle of 45° with respect to the x-axis. A further special case of interest occurs when $a_x = a_y$ and $\delta = \pm 90°$. The resulting wave is circularly polarized. When $\delta = +90°$, the wave is said to be left circularly polarized and, when $\delta = -90°$, it is said to be right circularly polarized. Thus, from (3.4) we have for $\delta = +90°$, at $\delta_y = 0$, $z = 0$ and $t = 0$, that $E_x = 0$ and $E_y = a_y$ as in Fig. 3.2a. Under the same conditions but at a later time such that $\omega t = 90°$, $E_y = 0$ and $E_x = a_x$, as shown in Fig. 3.2b. The rotation of the electric-field

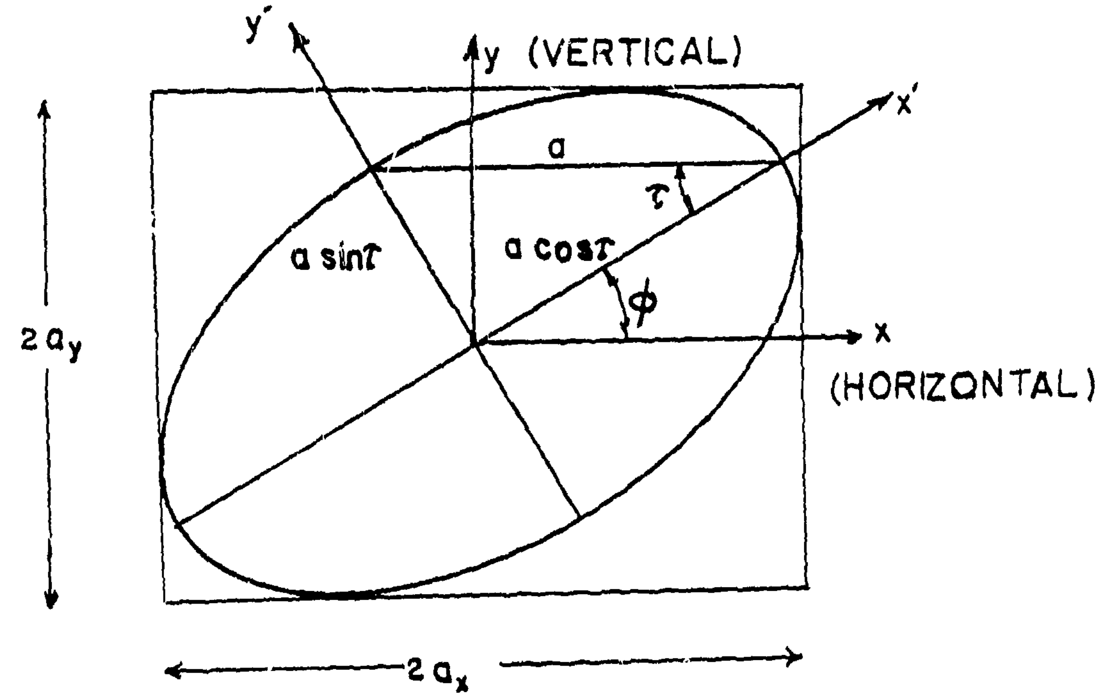


Fig. 3.1 : The Polarization Ellipse.

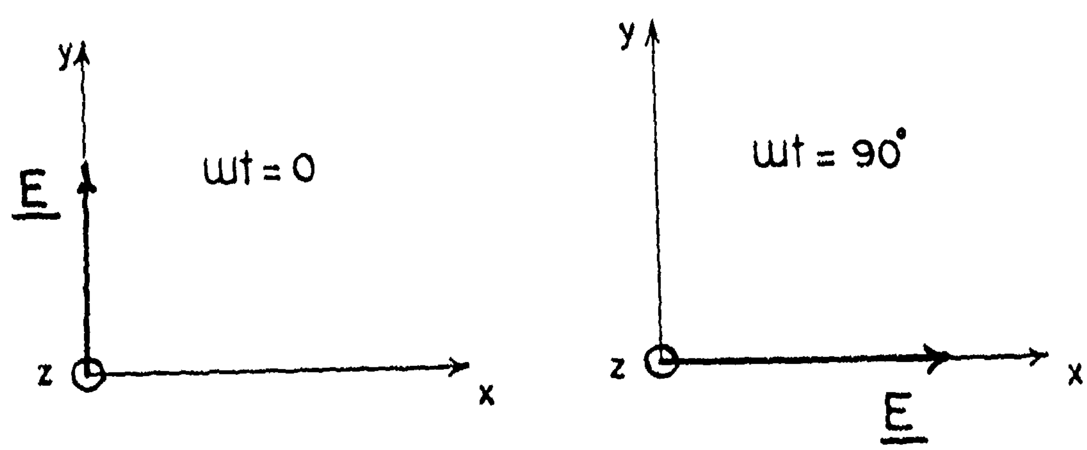


Fig. 3.2 : Change in direction of E for left circular polarization. Time wt=0 in (a) and wt=90 in (b) (Kraus 1966).

vector is thus clockwise with the wave approaching. According to the IEEE standards(1979) this sense of rotation is definded as left circular polarization. According to the older usage of classical physics (Born and Wolf), this sense of rotation (clockwise with wave approaching) is defined as right circular polarization, or opposite to the IEEE definition (Kraus 1966).

If the wave is viewed receding (from negative z axis in Fig.3-1), the electric vector appears to rotate in the opposite direction. Hence, clockwise rotation with the wave receding is the same as counterclockwise rotation with the wave approaching. In the following the IEEE definition will be used, since it could also be defined (without reference to the wave direction) by means of helical-beam antennas (Kraus, 1950). Thus, a right handed helical-beam antenna radiates or receives right circular (IEEE) polarization. A right-handed screw, is right handed regardless of the position from which the helix is viewed. There is no possibility here of ambiguity. In general Eq. (3.5) represents a left-handed elliptical polarized (ep) wave if $\sin \delta > 0$ and right-handed if $\sin \delta < 0$.

We now seek ultimately to define the polarization vector in terms of the geometric descriptors of the ellipse of the elliptical polarized (ep) plane wave it represents. The reason for this, is that, Eq. (3.3) is inconvenient when it comes to dealing with the proporties of radar targets since $\underline{h}$ is being associated with a fixed (xyz) antenna coordinate system of which the radar targets are independent.

The geometric descriptors of an ellipse are :

1) its size given by the magnitude $a = |\underline{h}|$ of the polarization vector,

2) the orientation of the ellipse with respect to the x-axis given by the orientation angle $\phi (0 \leq \phi \leq \pi)$,

3) the ellipticity angle τ, $(-\pi/4 \leq \tau \leq \pi/4)$ such that cot τ is given by the ratio of the semimajor to semiminor axes of the ellipse and it represents the axial ratio (AR), where $1 \leq AR \leq \infty$, and

4) the sense in which the ellipse is being traversed. It will be shown later that the sense can be given by the sign of τ. Though, it seems natural to define the sense as right-handed or left-handed according to whether the rotation of the electric vector $\underline{E}$ and the direction of propagation from a right-handed or left-handed screw as explained before.

We now choose a coordinate system (x',y') such that the ellipse has an orientation angle $\phi = 0$ in these coordinates. According to Fig. 3.1, the polarization vector $\underline{h}$ in this case can be given by :

$$\underline{h}(a,\tau,\alpha) = \begin{bmatrix} a\cos\tau \\ ja\sin\tau \end{bmatrix} e^{j\alpha} \tag{3.6}$$

where α is called the "absolute phase" of the antenna, and defines the phase reference of the antenna at time $t = 0$. From (3.6), we notice that, when the sign of τ changes, the direction of sense of polarization changes, being left-sensed for positive values of τ and right-sensed for the negative ones.

The general expression of the polarization vector for an elliptically polarized wave with an ellipse of orientation ϕ is obtained from (3.6) with the use of a rotation matrix :

$$\underline{h}(a,\phi,\tau,\alpha) = a\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\tau \\ j\sin\tau \end{bmatrix} e^{j\alpha} \tag{3.7}$$

The two descriptions of the polarization vector $\underline{h}$ of (3.3) and (3.7) are equivalent, except that in the case of (3.7), the antenna is descriped by geometric parameters. The parameters a, ϕ, τ, α are related to a_x, a_y, δ_x and δ_y according to the relations (Appendix A) :

$$a^2 = a_x^2 + a_y^2$$

$$\tan 2\phi = \frac{2a_x a_y}{a_x^2 - a_y^2} \cos\delta \tag{3.8}$$

$$\sin 2\tau = \frac{2a_x a_y}{a_x^2+a_y^2} \sin\delta$$

3.2.3 The Stokes Parameters

A practical way of representing the state of polarization of an elliptical wave is through a set of parameters, which all have the same physical dimensions. Such are the Stokes parameters and they were first introducted by Stokes in his studies of partially polarized light. These parameters constitute a column vector $\underline{g}$ (Stokes vector) and they can be defined in terms of the electric field vector components, or the set(a_x, a_y, δ_x, δ_y) or (a, ϕ, τ, α) as defined in Sections 3.2.1 and 3.2.2, in the following manner (Appendix B) :

$$\begin{aligned}
g_0 &= |h_x|^2+|h_y|^2 = a_x^2 + a_y^2 = a^2 &&= I \\
g_1 &= |h_x|^2-|h_y|^2 = a_x^2 - a_y^2 = a^2\cos 2\tau\cos 2\phi &&= Q \\
g_2 &= 2\mathrm{Re}\{h_x h_y^*\} = 2a_x a_y\cos\delta = a^2\cos 2\tau\sin 2\phi &&= U \\
g_3 &= -2\mathrm{Im}\{h_x h_y^*\} = 2a_x a_y\sin\delta = a^2\sin 2\tau &&= V
\end{aligned} \qquad (3.9)$$

where :

$$g_0^2 = g_1^2+g_2^2+g_3^2 = I^2 = Q^2+U^2+V^2 \qquad (3.10)$$

The component g_0 describes the intensity while g_1, g_2 and g_3 represent the polarization of an ep wave since they depend on the orientation of its ellipse and through the ellipticity angle τ on the sense in which the ellipse is being described. The four Stokes parameters have the dimensions of intensity(power); each corresponds not to an instantaneous intensity but to a time - averaged intensity, where the average is extended over an interval long enough to permit practical measurement. Although its components are physically real parameters, the Stokes vector is a mathematical vector, i.e. it is not defined in a three - dimensional physical space but in a four-dimensional mathemathical space. The description of polarization through the Stokes parameters is widely applicable since it covers the completely, partially or unpolarized waves. The physical property of the 4-component vector will be discussed in another forthcoming report dealing with quasi-coherent pan-chromatic wave interaction.

In practice the modified Stokes vector g_m is often used with components

$$g_{m0} = \tfrac{1}{2}(I+Q) = |h_x|^2 = a_x^2 = I_x$$
$$g_{m1} = \tfrac{1}{2}(I-Q) = |h_y|^2 = a_y^2 = I_y \qquad (3.11)$$

and

$g_{m2}=g_2$, $g_{m3}=g_3$ as given in Eq. (3.9)

3.2.4 The Poincare Sphere :

The Poincare sphere concept consists of mapping polarization states on points of a sphere for completely polarized ep waves. It constitutes a useful way of representing polarized ep waves as it will be shown as the present treatment unfolds. The polarization state of an ep wave, which either is described by the polarization vector $\underline{h}$ or the Stokas vector $\underline{g}$, can be represented on the Poincare sphere. Thus one can map the polarization vector $\underline{h}$ with complex components h_x and h_y, given by (3.3), on the Poincare sphere by using the auxiliary complex parameter

$$u = \frac{h_x - jh_y}{h_x + jh_y} = \frac{1-jP}{1+jP} \tag{3.12}$$

where P is the complex polarization ratio and it is equal to

$$P = \frac{h_y}{h_x} \tag{3.13}$$

Using (3.12) and (3.13), the polarization vector $\underline{h}$ can be represented uniquely on a point on the Poincare sphere with spherical coordinates (r, θ, ϕ') which are given by (Appendix C) :

$$r = |\,\underline{h}\,|^2 = a^2$$

$$\text{co-latitude } \theta = \arccos\{\frac{|u|^2-1}{|u|^2+1}\} = \pi/2-2\tau \tag{3.14}$$

$$\text{longitude } \phi' = 2\phi = -\text{phase}\{u\} = -\arctan\{\frac{\mathrm{Im}\{u\}}{\mathrm{Re}\{u\}}\}$$

It should be noted that absolute phases cannot be represented on the Poincare sphere. In case the polarization state is described by the Stokes vector $\underline{g}$, it is easily seen from Eqs.(3.9) that its g_1, g_2, g_3 components can be regarded as the cartesian coordinates of a point P on a sphere of radius g_0, such that $(\pi/2-2\tau)$ and 2ϕ are the spherical angular coordinates of this point(Fig.3.3).

According to the above properties, the various states of polarization can be mapped on the Poincare sphere which has the following properties(Fig.3.4) :

1. The Equatorial(x-y) plane divides the sphere into the left-sensed upper hemisphere where τ is positive $(0 < \tau \leq \pi/4)$ and the right-sensed lower hemisphere where τ is negative $(-\pi/4 \leq \tau < 0)$.

2. All linear polarizations $(\tau = 0°)$ are represented on the Equator with horizontal polarization(H) at zero longitude $\phi' = 2\phi = 0$ and the vertical polarization(V) at the antipodal location $\phi' = 2\phi = \pi$

3. Left-handed circular polarization(LC,$\tau=\pi/4$) is mapped on the zenith $(g_1=g_2=0, g_3=g_0)$ and right-handed circular polarization (RC,$\tau=-\pi/4$) on the nadir $(g_1=g_2=0, g_3=-g_0)$.

4. Any two orthogonal polarizations $\underline{h}\,(a,\phi,\tau,\alpha)$ and $\underline{h}_\perp(a_\perp,\phi+\pi/2,-\tau,\alpha_\perp)$ are mapped on antipodal points.

5. Statistical polarization can also be mapped on the Poincare sphere, thermal radiation produces random polarization states uniformly distributed over the polarization sphere.

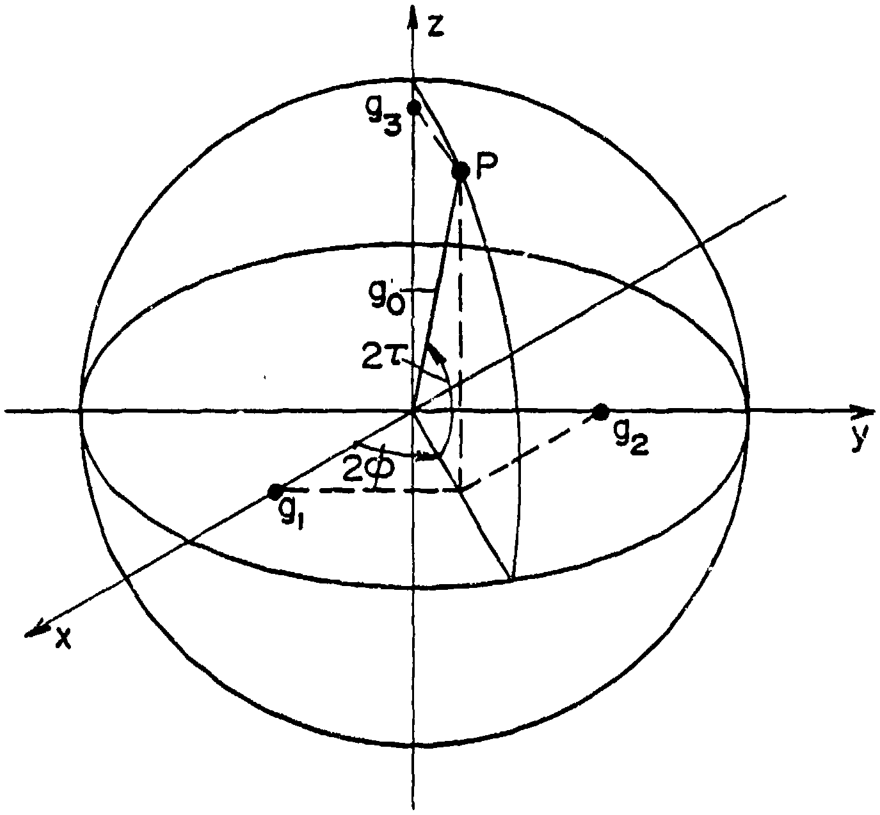


Fig. 3.3 : Representation of a polarization vector defined by its Stokes vector $\underline{g}=(g_0, g_1, g_2, g_3)$ on the Poincare' sphere.

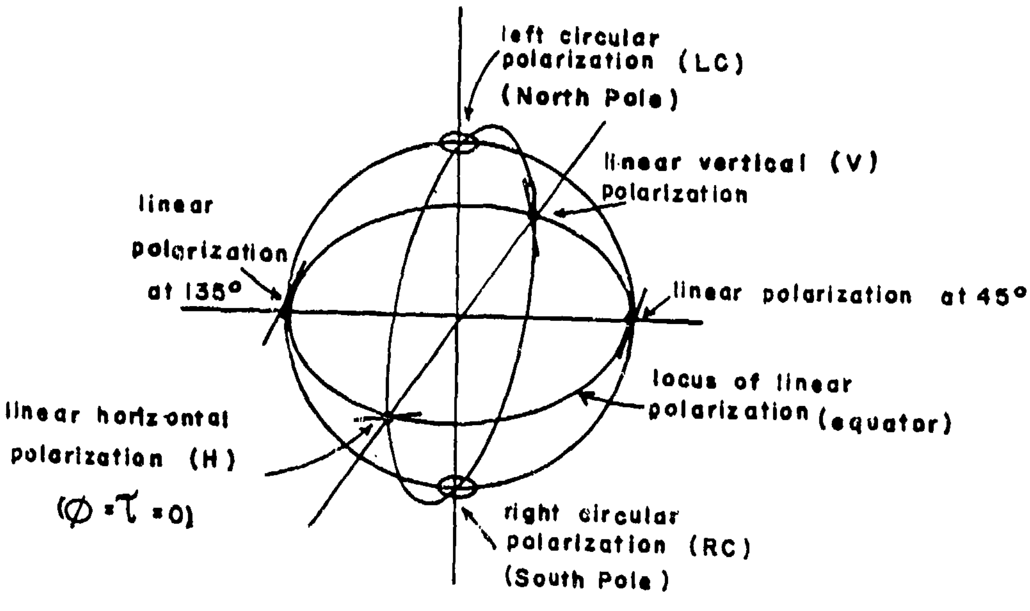


Fig. 3.4 : Polarization at Cardinal Points of Poincare' sphere (Kraus 1966).

3.2.5 Polarization Charts :

Polarization charts provide a useful tool for a simplified representation of the state of any polarization vector on a two-dimensional chart instead of using the three-dimensional Poincare sphere, which though is still preferable in case the XPOL and COPOL null characteristics need to be studied in detail. Deschamps(1951,1953) in his tutorial study on the Hyperbolic Protractor and Rumsey(1951) showed how various mappings on the sphere can be related to the power impedance(Smith) chart which was further followed up by Huynen(1960), Poelman(1971), and also in the Russian literature as e.g. in Kanareykin et al(1966,1968). There exist many types of polarization charts and here we will briefly review the properties of some charts which are being used most frequently.

1. Rumsey p-and q-Charts :

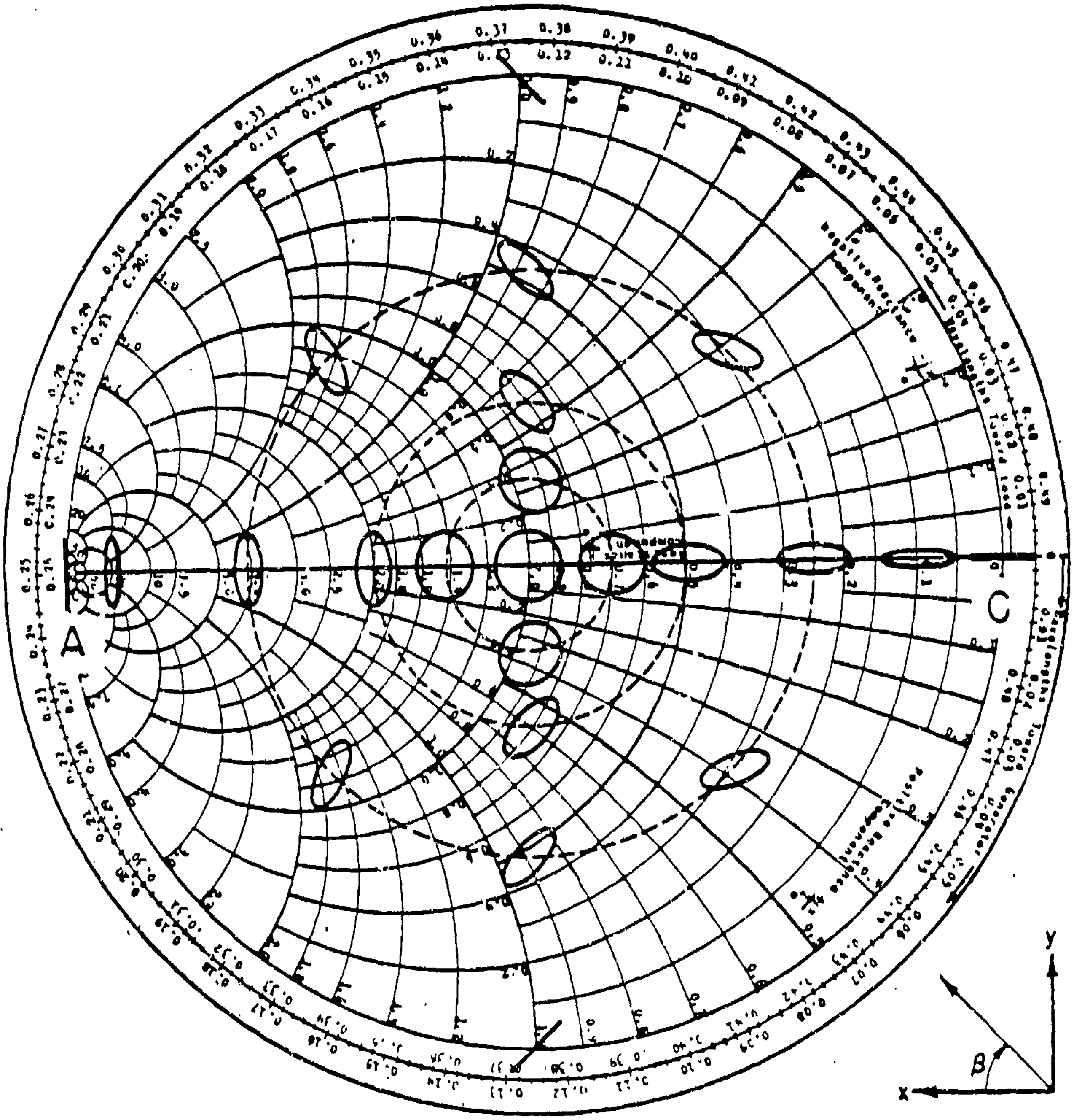
Rumsey(1951) has used the impedance concept which reduces the field problem to a transmission line problem. The analysis of impedance transformation occuring in transmission line theory can again be simplified by working in terms of reflection coefficients. The impedance concept is successful in such applications largely because the tangential components of the electric and magnetic fields are continuous at a surface separating two different media.

For the same reason, he used the polarization ratio P defind as the ratio of two orthogonal tangential components of electric field, to be equally valuable in the analysis of polarization.The polarization ratio P is analogous to impedance, then Rumsey finds that the analogue of the reflection coefficient is the ratio of the right-handed and left-handed circularly polarized components which are equivalent to the linearly polarized components used to define P. Following Rumsey(1951) :

$$q = \frac{1-p}{1+p} , \quad p = \frac{1-q}{1+q} , \quad p = \frac{jh_y}{h_x} = jP , \quad q = \frac{h_\ell}{h_r} ,$$

h_x, h_y are the linear x and y components of the polarization vector, and h_r, h_ℓ are the right and left circularly polarized components. Note that p=jP =j(Polarization ratio). The transformation from p to q is thus identical to the transformation from the current reflection coefficient to the normalized impedance. In view of the symmetry of the transformation we can think of p, and vice versa q, as reflection coefficient or impedance. By using these relations, Rumsey has developed the q and p charts for representing any polarization vector.

Fig.3.5 shows how the orientation and shape of the polarization ellipse are represented using th Smith-Buschbeck Chart as the q-plane. Fig.3.6 illustrates the representation using the Carter-Schmidt diagram as the p-plane.



q Plane Representation

Fig. 3.5 : q-plane Polarization Chart (Rumsey 1951).

For points on the q-plane Fig.3.5, (Rumsey (1951)) :

a) We have right-handed polarization for all points inside the unit circle, and left-handed polarization for all points outside.

b) The origin represents right-circular polarization, and the point at infinity represents left-circular polarization.

c) Constant axial-ratio contours are identical to the circles of constant SWR on a Smith chart. The axial ratio is equal to the SWR obtained by treating q as a reflection coefficient.

d) A point on the unit circle represents linear polarization at an angle equal to one-half the polar angle.

e) The angle ϕ between the x axis and the major axis of the polarization ellipse is related to the polar angle θ on the plane by the simple relation $2\phi=\theta$.

f) the locus of the points representing polarizations for which $|h_x/h_y|$ is constant, is the set of circles passing through the points $q=\pm1$ (the short and open-circuit points), the orthogonal set of circles is obtained if the phase of h_x/h_y is constant.

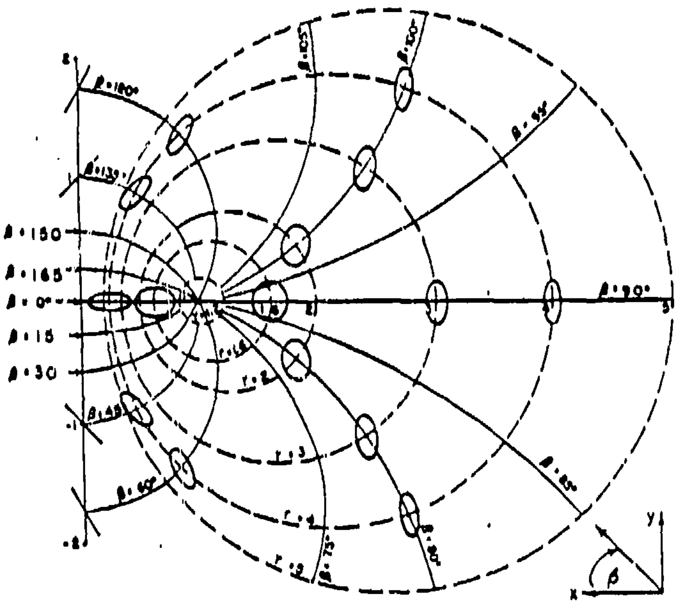
For points on the p-plane, Fig.3.6, (Rumsey (1951)) :

a) Points in the right half-plane represent right-handed polarization and points in the left-plane represent left-handed polarization.

b) Points on the vertical axis represent linear polarization at various angles.

c) Circular polarization is represented by the points(1,0) and (-1,0).

d) Constant axial-ratio contours are identical to the circles of constant standing wave ratio(SWR) on the Carter-Schmidt impedance chart. The axial ratio is equal to the SWR obtained by treating p as an impedance.



p Plane Representation

Fig. 3.6 : p-plane Polarization Chart (Rumsey 1951).

2. Deschamps O and S charts :

Deschamps(1951,1953) has used the polarization ratio P which can be deduced from the parameters ϕ, τ(orientation and ellipticity angles), to derive the two sets of equations

$$\cos 2\gamma = \cos 2\tau \cos 2\phi$$

$$\tan \delta = \tan 2\tau \csc 2\phi$$

or

$$\tan 2\phi = \tan 2\gamma \cos \delta$$

$$\sin 2\tau = \sin 2\gamma \sin \delta$$

where $P = \tan\gamma e^{j\delta}$ (δ is the phase difference between y and x channels and $\tan\gamma$ is the amplitude of the polarization ratio). He has used two projections on the equator of the sphere. The first one, an orthographic projection (chart O) is shown in Fig.3.7. The other, a stereographic projection from the nadir, is shown on Fig.3.8 (Chart S).

In both cases by drawing the lines along which ϕ and τ are constant (meridians and parallels) and the lines along which δ and γ, or the ratio $\tan\gamma$, are constant, we have a method of direct conversion from any two of these parameters to the others.

The lines δ constant are circles on Chart S and ellipses on Chart O. The lines γ constant are straight lines perpendicular to HV on Chart O and circles on Chart S.

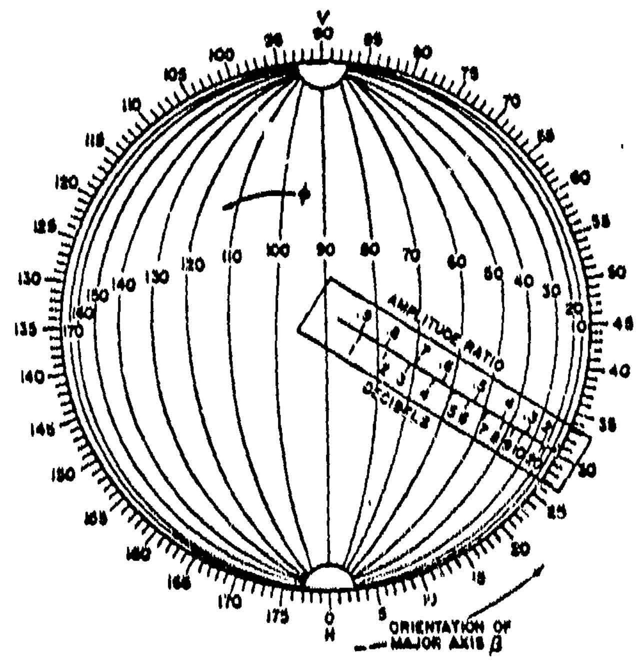


Fig. 3.7 : Orthographic Polarization Chart(O) (Deschamps 1951,53).

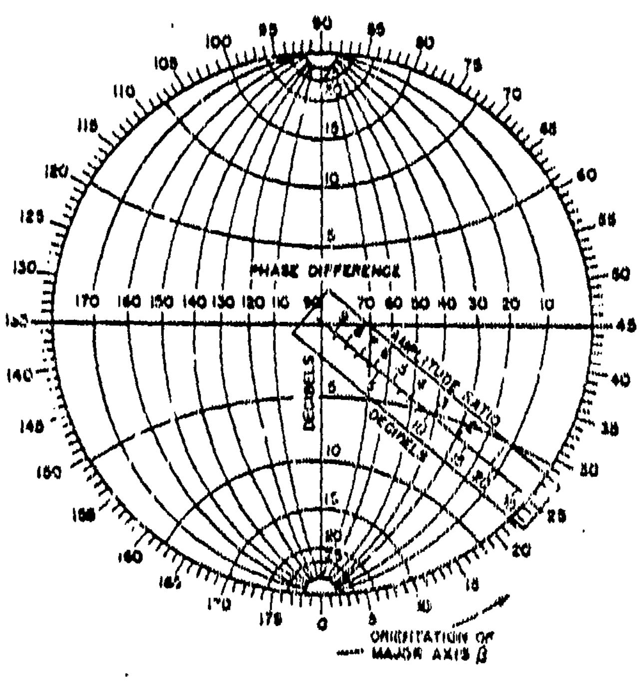


Fig. 3.8 : Stereographic Polarization Chart(S) (Deschamps 1951,53).

3. Huynen's Chart :

Huynen(1970) has presented two polarization charts, each representing one hemisphere (top and bottom), to map the whole sphere on a plane. Fig.3.9 shows such a circular polarization chart, which maps all positive or left-sensed polarizations. The circumference of the circular chart gives all linear polarizations and the chart is left circular. Notice the effect of 2ϕ on points of the chart such that "horizontal" polarization ($\phi=0°$) is mapped on the extreme right- hand side of the chart, while "vertical" polarization ($2\phi=180°$) is mapped on the extreme left-hand side. All points on the chart represent polarization with orientation $\phi=45°$. Note the interesting fact that the radial distance of a point on the polarization chart is measured by $\cos 2\tau$.

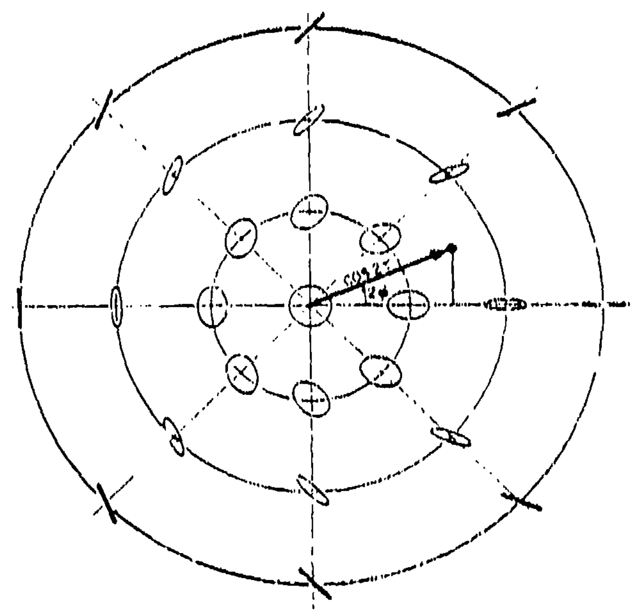


Fig. 3.9 : Huynen's Polarization Chart (Huynen 1970).

4. Poelman's Modified Chart :

Poelman(1971) has modified Huynen's chart which was explained before. He used in his modified polarization chart the polar coordinates 2ϕ and r such that :

$$r = \tan\tau = \pm\sqrt{\frac{1-\cos 2\tau}{1+\cos 2\tau}}$$

as then the geometric parameters ϕ and r can easily be read off. Note that Poelman used $r=\tan\tau$ instead of $\cos 2\tau$ which was used by Huynen.

In Fig.3.10 the distinguished polarizations with the proposed code numbers for the right-sensed polarization are presented on the modified polarization chart. A similar chart for the left-sensed polarization can be given, where for the linear polarization the same code numbers can be taken as given in Fig.3.10.

In Table 3.1, the 149 used polarizer settings (θ,δ) for the right-sensed polarizations, including the 28 linear polarization and the corresponding code numbers are given. It follows that a group of 270(149+149-28) different polarizations (right-and-left-sensed) is chosen to represent all possible radar polarizations.

In Table 3.2 the polarization characteristics (r and ϕ) and corresponding code numbers are listed for the right-sensed

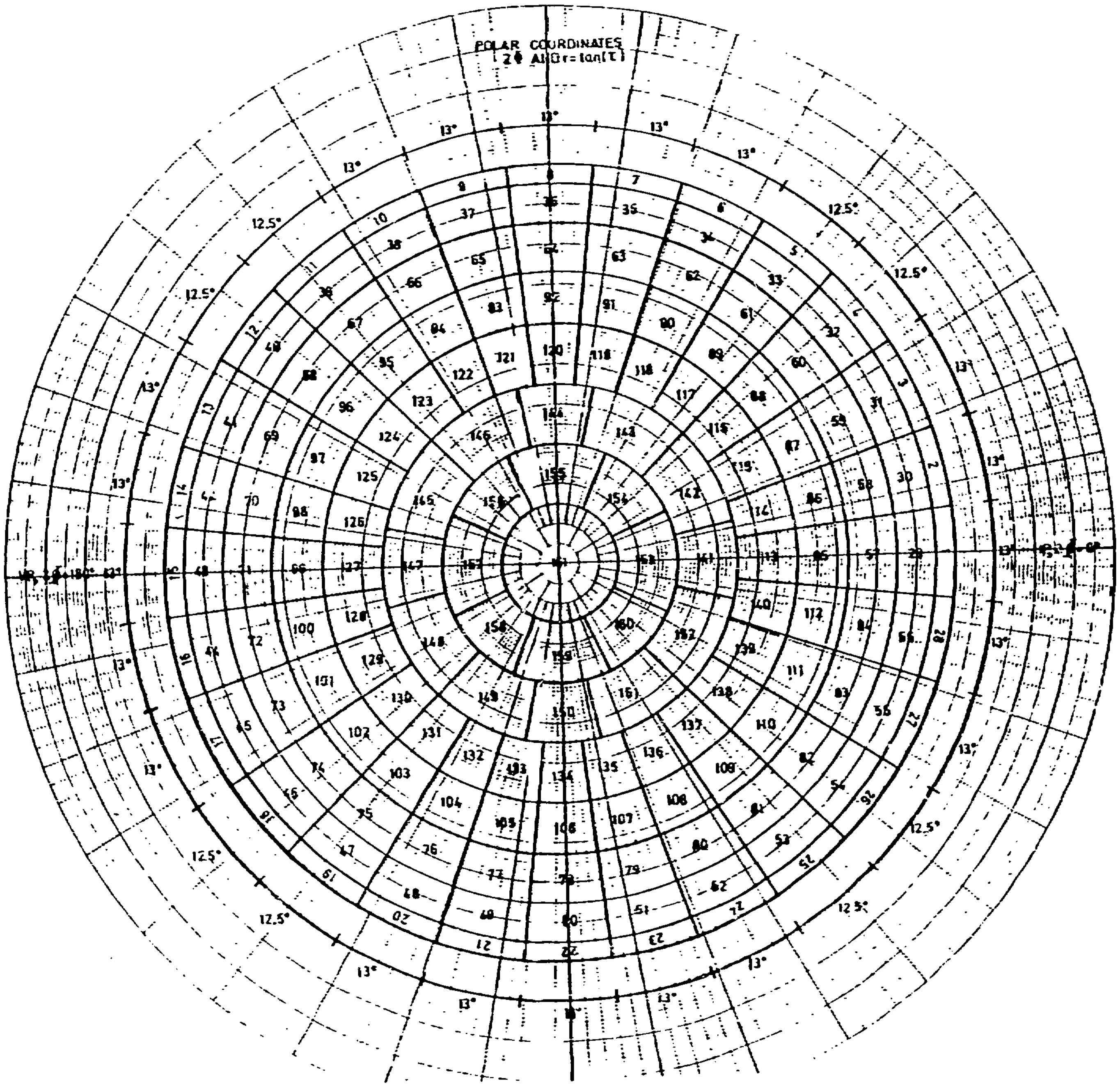


Fig. 2.10. Poelman's modified Polarization Chart (a) (Poelman 1971).

δ / θ	0°	13°	26°	39°	51°	64°	77°	90°	103°	116°	129°	141°	154°	167°	180°
0°	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13°	2	-	-	30	-	-	-	29	-	-	-	56	-	-	28
26°	3	-	31	-	-	58	-	57	-	84	-	-	55	-	27
39°	4	32	-	59 60	-	87	86	85	112	111	-	82 83	-	54	26
51°	5	33	61	-	88	115	114	113	140	139	110	-	81	53	25
64°	6	34	62	89 90	117	116	142	141	152	138	137	108 109	80	52	24
77°	7	35	63	91	118 119	143	154	153	160	151	135 136	107	79	51	23
90°	8	36	64	92	120	144	155	$\boxed{161}$	159	150	134	106	78	50	22
103°	9	37	65	93	121 122	145	156	157	158	149	132 133	105	77	49	21
116°	10	38	66	94 95	123	124	146	147	148	130	131	103 104	76	48	20
129°	11	39	67	-	96	125	126	127	128	129	102	-	75	47	19
141°	12	40	-	68 69	-	97	98	99	100	101	-	73 74	-	46	18
154°	13	-	41	-	-	70	-	71	-	72	-	-	45	-	17
167°	14	-	-	42	-	-	-	43	-	-	-	44	-	-	16
180°	-	-	-	-	-	-	-	-	-	-	-	-	-	-	15

Table 3.1: Survey of the 149 used right-sensed polarizations; the polarization characteristics corresponding with the 161 code numbers are presented in Table 3.2, and the areas they represent on the polarization chart are given in Fig. 3.10 (Poelman, 1971).

code No.	r	Φ	code No.	r	Φ	code No.	r	Φ
1	0	0°	19	0	115.5°	37	0.11	51.7°
2	0	6.5°	20	0	122°	38	0.10	58.3°
3	0	13°	21	0	128.5°	39	0.09	64.9°
4	0	19.5°	22	0	135°	40	0.07	70.9°
5	0	25.5°	23	0	141.5°	41	0.10	78.2°
6	0	32°	24	0	148°	42	0.07	84.9°
7	0	38.5°	25	0	154.5°	43	0.11	90°
8	0	45°	26	0	160.5°	44	0.07	95.1°
9	0	51.5°	27	0	167°	45	0.10	101.8°
10	0	58°	28	0	173.5°	46	0.07	109.1°
11	0	64.5°	29	0.11	0°	47	0.09	115.1°
12	0	70.5°	30	0.07	5.1°	48	0.10	121.7°
13	0	77°	31	0.10	11.8°	49	0.11	128.3°
14	0	83.5°	32	0.07	19.1°	50	0.11	135°
15	0	90°	33	0.09	25.1°	51	0.11	141.7°
16	0	96.5°	34	0.10	31.7°	52	0.10	148.3°
17	0	103°	35	0.11	38.3°	53	0.09	154.9°
18	0	109.5°	36	0.11	45°	54	0.07	160.9°

Table 3.2: Survey of the polarization characteristics ellipticity ratio, r, and orientation angle, Φ, corresponding with the code numbers for the right-sensed polarizations. (Poelman, 1971)

code No.	r	Φ	code No.	r	Φ	code No.	r	Φ
55	0.10	168.2°	76	0.21	120.7°	97	0.31	80.2°
56	0.07	174.9°	77	0.23	127.8°	98	0.34	84.8°
57	0.23	0°	78	0.23	135°	99	0.35	90°
58	0.21	6.0°	79	0.23	142.2°	100	0.34	95.2°
59 60	0.21	16.1°	80	0.21	149.3°	101	0.31	99.8°
61	0.18	24.0°	81	0.18	156.0°	102	0.34	108.9°
62	0.21	30.7°	82 83	0.21	163.9°	103 104	0.31	118.9°
63	0.23	37.8°	84	0.21	174.0°	105	0.34	126.8°
64	0.23	45°	85	0.35	0°	106	0.35	135°
65	0.23	52.2°	86	0.34	5.2°	107	0.34	143.2°
66	0.21	59.3°	87	0.31	9.8°	108 109	0.31	150.1°
67	0.18	66.0°	88	0.34	18.9°	110	0.34	161.1°
68 69	0.21	73.9°	89 90	0.31	28.9°	111	0.31	170.2°
70	0.21	84.0°	91	0.34	36.8°	112	0.34	174.8°
71	0.23	90°	92	0.35	45°	113	0.48	0°
72	0.21	96.0°	93	0.34	53.2°	114	0.46	7.8°
73 74	0.21	106.1°	94 95	0.31	61.1°	115	0.41	14.2°
75	0.18	114.0°	96	0.34	71.1°	116	0.51	21.0°

Table 3.2 (con't)

code No.	r	Φ	code No.	r	Φ	code No.	r	Φ
117	0.41	26.1°	134	0.48	135°	149	0.59	121.1°
118 119	0.46	33.9°	135 136	0.46	146.1°	150	0.62	135°
120	0.48	45°	137	0.41	153.9°	151	0.59	148.9°
121 122	0.46	56.1°	138	0.51	159°	152	0.59	167.6°
123	0.41	63.9°	139	0.41	165.8°	153	0.80	0°
124	0.51	69.0°	140	0.46	172.2°	154	0.72	22.1°
125	0.41	75.8°	141	0.62	0°	155	0.80	45°
126	0.46	82.2°	142	0.59	12.4°	156	0.72	67.9°
127	0.48	90°	143	0.59	31.1°	157	0.80	90°
128	0.46	97.8°	144	0.62	45°	158	0.72	112.1°
129	0.41	104.2°	145	0.59	58.9°	159	0.80	135°
130	0.51	110.0°	146	0.59	77.6°	160	0.72	157.9°
131	0.41	116.1°	147	0.62	90°	161	1	-
132 133	0.46	123.9°	148	0.59	102.4°	-	-	-

Table 3.2 (con't)

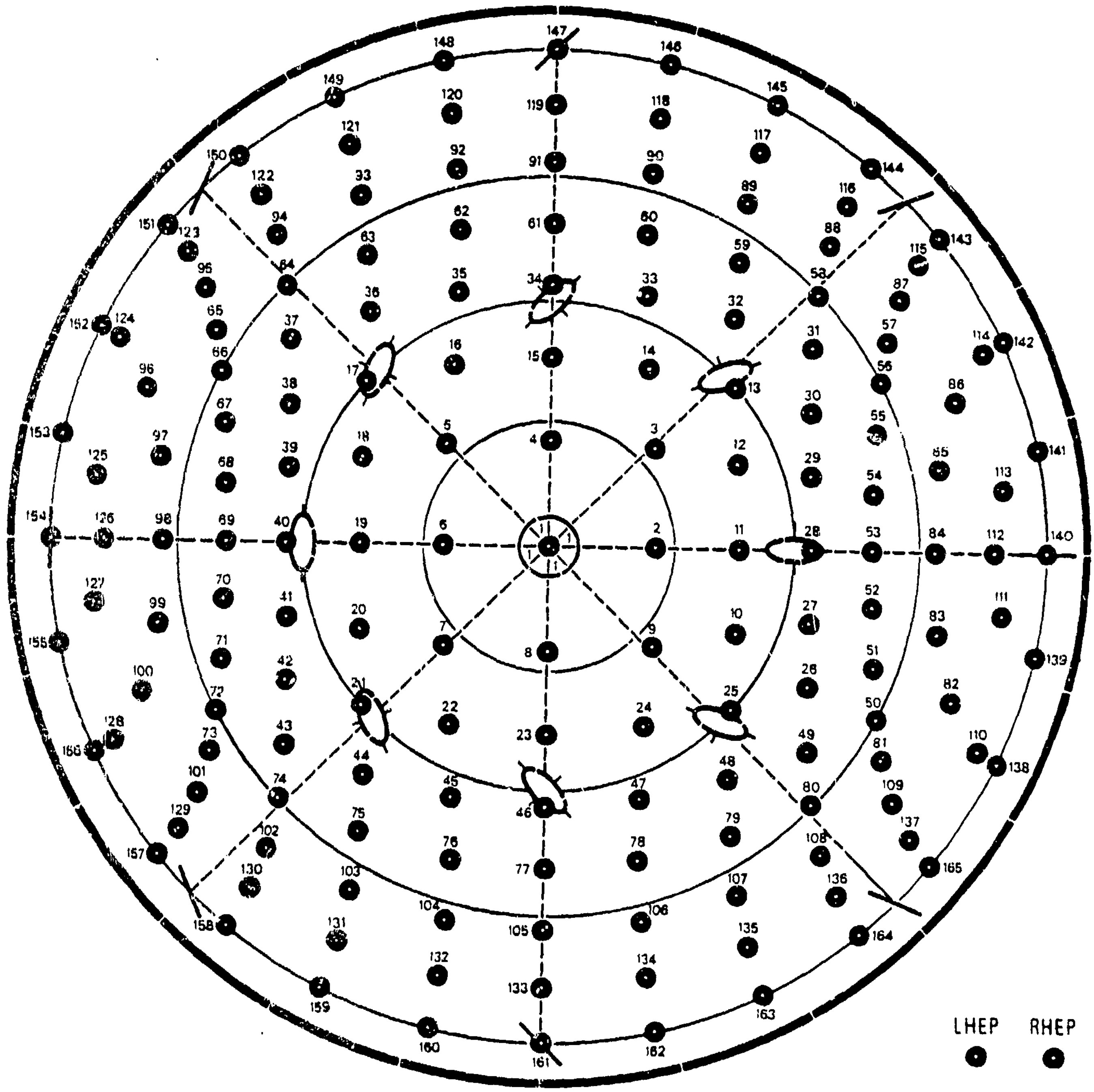


Fig. 3.11 : Poelman's modified Polarization Chart (b) (Poelman 1977,78).

polarization, including the linear polarization.

Poelman(1977,1978) has used also another version of the modified polarization chart Fig.3.11 which is very similar to the one shown in Fig.3.11 but with using the polar coordinates 2ϕ and $\rho=1-r$.

5. Optimal Polarization Charts :

The optimal polarization charts (Kennaugh, 1952, Huynen, 1970) will be discussed in the forthcoming reports. On these charts, the COPOL and XPOL nulls will be drawn for different aspect angles and different frequencies for various targets and sea clutter.

3.2.6 Canonical Polarization Pairs

The power $P(\underline{h}_j,h_i)$ received, by a receiving antenna with polarization $\underline{h}_j(a_j,\alpha_j,\phi_j,\tau_j)$ due to that transmitted by $\underline{h}_i(a_i,\alpha_i,\phi_i,\tau_i)$ is found to be given in general by (Huynen 1970):

$$P(\underline{h}_j,h_i) = \tfrac{1}{2}a_i^2a_j^2[1+\sin2\tau_j\sin2\tau_i+\cos2(\phi_j+\phi_i)\cos2\tau_j\cos2\tau_i] \quad (3.15)$$

Thus by selecting the parameters of the polarization pairs $(\underline{h}_j,h_i)$, one can determine according to (3.15) the magnitude of the received power. There are four such canonical polarization pairs (Fig.3.12) of special importance since they give rise to situations frequently occuring in practical transimission reception interactions. These are :

ORTHOGONAL : $\underline{h}_j = \{ \underline{h}_i\}^*_\perp$

$\underline{h}_{orthogonal}=\underline{h}_j(a_j$ and α_j arb.$,\phi_j=\phi_i+\frac{1}{2}\pi,\tau_j=-\tau_i)$,

antipodal, but undetermined in a and α:

CANNOT BE USED AS A UNIQUE DESCRIPTOR.

TRANSVERSE : $\underline{h}_j=\underline{h}_i(-\alpha_i,-\phi_i)$: $P(\underline{h}_j,\underline{h}_i)=P_{max}=a_i^2a_j^2$

$\underline{h}_{transverse}=\underline{h}_j(a_j=a_i,\alpha_j=-\alpha_i,\phi_j=-\phi_i,\tau_j=\tau_i)$:

OPTIMAL RECEPTION : ANTENNA MATCHING.

SYMMETRIC : $\underline{h}_j=\underline{h}_i(-\phi_i-\tau_i)$:$P(\underline{h}_j,\underline{h}_i)=a_i^4\cos^2 2\tau_i$

$\underline{h}_{symmetric}=\underline{h}_j(a_j=a_i,\alpha_j=\alpha_i,\tau_j=-\tau_i,\phi_j=-\phi_i)$:

MOST FREQUENCY TARGET POLARIZATION.

CONJUGATE : $\underline{h}_j=\underline{h}_i(-\alpha_i,-\tau_i)$

$\underline{h}_{conjugate}=\underline{h}_j(a_j=a_i,\alpha_j=-\alpha_i,\phi_j=\phi_i,\tau_j=-\tau_i)$:

PRECIPITATION (CIRCULAR CLUTTER).

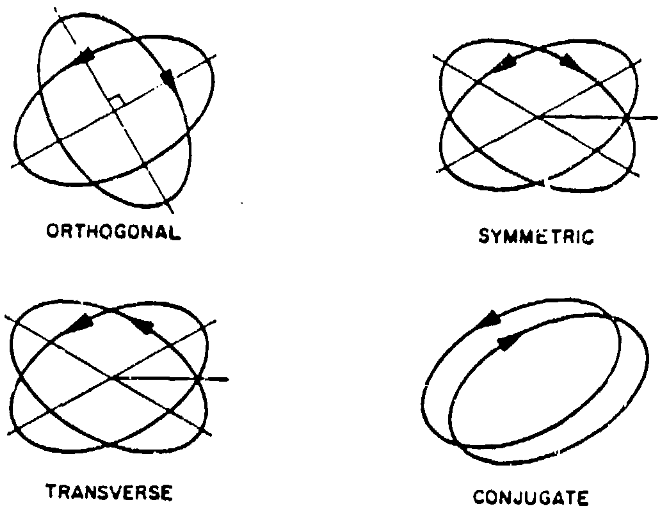


Fig. 3.12 : Basic Polarization Pairs.

3.3 Scattering Matrix[S] :

The general radar target detection technique consists of having an electromagnetic radiation originating from a radar transmitter scattered by the object under detection and sampled by the radar receiver. The measure of the intensity of the scattered radiation in the far zone from the scattering obstacle was usually described by the scalar radar range equation expressed in terms of the radar cross-section (RCS) σ given by :

$$\sigma=\lim_{R\to\infty}4\pi R^2\frac{|\underline{E}^s(\theta_s,\phi_s)|^2}{|\underline{E}^i(\theta_i,\phi_i)|^2}=\lim_{R\to\infty}4\pi R^2\frac{|\underline{H}^s(\theta_s,\phi_s)|^2}{|\underline{H}^i(\theta_i,\phi_i)|^2} \tag{3.16}$$

where R is the range (distance from target where the scattered radiation is observed, Fig.3.13), $\underline{E}^s$, $\underline{H}^s$ are the scattered electric and magnetic field vectors at the observation point at direction (θ_s,ϕ_s), and similarly $\underline{E}^i$, $\underline{H}^i$ define the incident electric and magnetic field vectors with direction angles θ_i, ϕ_i.

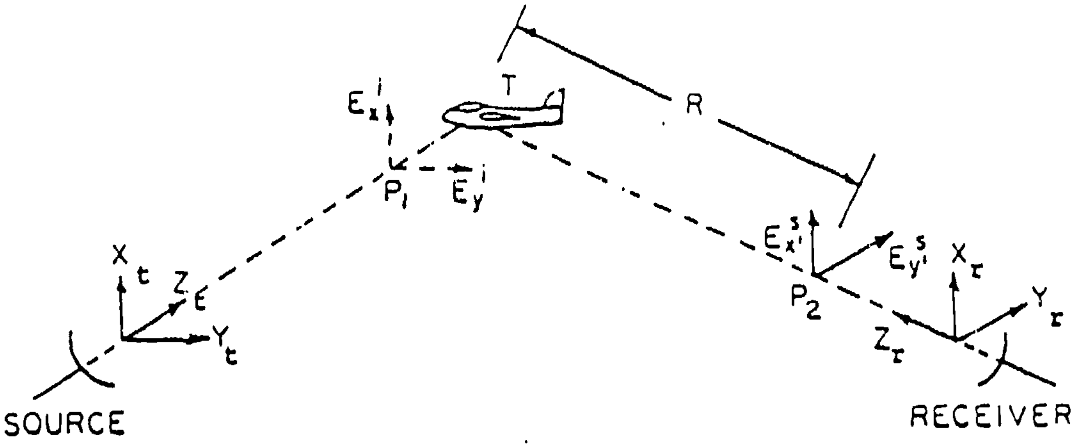


Fig. 3.13 : Illustration of Radar Range Equation.

Eq.(3.16) gave unsatisfactory results with clutter precipitation present thus yielding limited radar target detection. Recognizing these difficulties, Sinclair(1948) showed that by introducing polarization in the radar range equation the problems could be overcome. He proved that a radar target acts a polarization transformer and this polarization transformation can be described by a matrix known as the scattering matrix. We note that in Optics Jones(1941) introduced a similar matrix(Shurcliff, 1962).

Let the polarization vectors $\underline{h}^s$ and h^i for the scattered and incident radiation, respectively, be defined in terms of general orthogonal polarization base vectors $(\hat{h}_A, \hat{h}_B)$ i.e.

$$\underline{h}^s = \begin{bmatrix} h_A{}^s \\ \\ h_B{}^s \end{bmatrix} \quad \text{and} \quad \underline{h}^i = \begin{bmatrix} h_A{}^i \\ . \\ h_B{}^i \end{bmatrix} \tag{3.17}$$

The complex components h_A, h_r can represent any polarization by relative magnitudes and phases. The scattering matrix transforms the transmitted polarization $\underline{h}^i$ into the polarization of the scattered field $\underline{h}^s$ according to :

$$\underline{h}^s(\theta_s,\phi_s)=[\sqrt{\sigma}(\theta_s,\phi_s,\theta_i,\phi_i)]\underline{h}^i(\theta_i,\phi_i) \tag{3.18}$$

where $[\sqrt{\sigma}]$ in case of A=H, B=V is the "linear polarization" restricted scattering matrix with absolute phase, i.e.

$$[\sqrt{\sigma}(\theta_s,\phi_s;\theta_i,\phi_i)]=\frac{e^{j\phi_{AB}}}{\sqrt{4\pi R^2}}\begin{bmatrix} \sqrt{\sigma_{AA}}e^{j(\phi_{AA}-\phi_{AB})} & \sqrt{\sigma_{AB}} \\ \sqrt{\sigma_{BA}}e^{j(\phi_{BA}-\phi_{AB})} & \sqrt{\sigma_{BB}}e^{j(\phi_{BB}-\phi_{AB})} \end{bmatrix} \tag{3.19}$$

where σ_{ij} and ϕ_{ij} represent the radar cross-sections and the absolute phases of the target returns with received (i) and transmitted (j) polarizations. The matrix [$\sqrt{\sigma}$] in Eq.(3.19) is defined in terms of the absolute phase ϕ_{AB}.

By making the substitution :

$$\lim_{R\to\infty} \frac{\sqrt{\sigma_{ij}}}{\sqrt{4\pi R^2}} = |S_{ij}| \quad \text{where } i,j\text{=A or B} \tag{3.20}$$

then Eq.(3.18) become :

$$\underline{h}^s = [S]_{SMA}\underline{h}^i \tag{3.21}$$

where $[S]_{SMA}$ is the scattering matrix with absolute phase and it can be written from (3.19) and (3.20) by :

$$[S]_{SMA} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} = e^{j\phi_{AB}} \begin{bmatrix} |S_{AA}|e^{j(\phi_{AA}-\phi_{AB})} & |S_{AB}| \\ |S_{BA}|e^{j(\phi_{BA}-\phi_{AB})} & |S_{BB}|e^{j(\phi_{BB}-\phi_{AB})} \end{bmatrix} \tag{3.22}$$

The scattering matrix represents the radar target for a given frequency and fixed aspect angle . Thus at these fixed frequency and aspect angle, the target is described completely by the scattering matrix. However, in special cases one and the same matrix may represent properties of a set of different targets (Huynen, 1970). A particularly useful feature of the scattering matrix representation is that its intrinsic properties are a function of the target configuration and do not depend on the measurement technique or measuring equipment. However, the scattering matrix with absolute phase does depend on the target displacement along the line of sight. In order for the $[S]_{SMA}$ matrix to be specified completely, eight real numbers(four magnitudes and four phases) have to be measured. The measurement of the absolute phase of a target becomes very formidable because of its dependence on the location of the target, the direction of illumination, surface configuration and the radar frequency. Hence, one has to use highly elaborate measuring techniques. Moreover, the absolute phases cannot be mapped on the Poincare sphere (Thiel 1970) and thus the $[S]_{SMA}$ matrix cannot have a unique representation on the sphere. Because of these difficulties, the scattering matrix with relative phase(SMR), where any one of the phases of the matrix elements can be set to an arbitrary constant (most commonly the phase of the cross-polarized term S_{AB}, i.e. ϕ_{AB} is chosen to be zero). Thus the scattering matrix with relative phase $[S]_{SMR}$ is according to (3.22) :

$$[S]_{SMR} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \text{ with } \phi_{AA}, \phi_{BA}, \phi_{BB} \neq 0, \text{ and } \phi_{AB} = 0 \qquad (3.23)$$

This simplification reduces the number required to specify completely the scattering matrix from eight to seven real numbers.

Another important feature of defining relative phases instead of the absolute ones is that relative phase terms can be recovered from amplitude only data (Kennaugh, 1949). In the case of "bistatic" scattering one has : $\theta_s \neq \theta_i$, $\phi_s \neq \phi_i$ and $S_{AB} \neq S_{BA}$. In the "monostatic" case (both transmitter and receiver antennas are at the same location), we have $S_{AB}=S_{BA}$ because the reciprocity theorem and conservation of energy should be satisfied for propagation in an isotropic medium. We note that for anisotropic scatters this condition need not be satified. Thus the scattering matrix$[S]_{SMR}$ becomes symmetric in the "monostatic" case and only five real numbers(three amplitudes and two phases) are needed to specify it completely.

Using (3.17), (3.21) and (3.22), we have

$$\begin{bmatrix} h_A{}^s \\ h_B{}^s \end{bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \begin{bmatrix} h_A{}^i \\ h_B{}^i \end{bmatrix} \tag{3.24}$$

The polarization of the electromagnetic wave in (3.24) is defined with respect to two orthogonal components which are in general elliptical. The two special cases of elliptical polarization i.e. linear and circular polarizations are the most commonly used for reference systems. If we represent the linear polarization vector with (h_x, h_y), and the circular polarization vector with (h_r, h_ℓ) as its right and left circular components, respectively, then the relations between the scattered fields in the two polarizations are given by (Long 1966) :

$$h_{\ell} = \tfrac{1}{2}(h_x+jh_y),\ h_r = \tfrac{1}{2}(h_x-jh_y) \qquad (3.25)$$

The scattering matrices for the two polarizations are related through the formulas :

$$C_{\ell\ell} = |\tfrac{1}{2}(S_{xx}-S_{yy})+jS_{xy}|,\ S_{xy}=S_{yx}$$

$$C_{\ell r} = |\tfrac{1}{2}(S_{xx}+S_{yy})|,\ C_{\ell r}=C_{r\ell} \qquad (3.26)$$

$$C_{rr} = |\tfrac{1}{2}(S_{xx}-S_{yy})-jS_{xy}|$$

where the C_{ij} are the elements of the scattering matrix with respect to the circular polarization basis and S_{ij} are the corresponding elements with respect to linear polarization basis.

3.4 Scattering Matrix Transformation Invariants :

3.4.1 The unitary transformation matrix [T] :

Assuming reciprocity holds, there exists an infinite number of general pairs of orthogonal elliptical polarization basis vectors $\hat{h}_A, \hat{h}_B$ and an infinite number of possible invariant transformations. Numerically, the transformation proporties of the scattering matrix [S(A,B)] (assuming no polarization losses) from one orthogonal pair :

$$\underline{h} = h_A\hat{h}_A + h_B\hat{h}_B \qquad (3.27)$$

to another orthogonal pair :

$$\underline{h} = h_A'\hat{h}_A' + h_B'\hat{h}_B' \qquad (3.28)$$

can be transformed to another new scattering matrix $[S'(A',B')]$ by using a unitary transformation matrix $[T]$. This matrix $[T]$ relates the polarization vector components in the two orthogonal basis sets (A,B) and (A',B'). To find the matrix [T], the orthogonality relationship between the components of each of the two orthogonal basis vectors should be used e.g.

$$\hat{h}_i \cdot \hat{h}_j^{\,*} = \delta_{ij} \tag{3.29}$$

where δ_{ij} is the Kronecker delta function ($\delta_{ij}=1, i=j, \delta_{ij}=0, i\neq j$) and (i,j) may be equal to A or B in the first basis set and A' or B' in the second set.

From (3.27) to (3.29), we have :

$$\underline{h} \cdot \hat{h}_A^{\,*} = h_A = h_A'(\hat{h}_A' \cdot \hat{h}_A^{\,*}) + h_B'(\hat{h}_B' \cdot \hat{h}_A^{\,*}) \tag{3.30}$$

and

$$\underline{h} \cdot \hat{h}_B^{\,*} = h_B = h_A'(\hat{h}_A' \cdot \hat{h}_B^{\,*}) + h_B'(\hat{h}_B' \cdot \hat{h}_B^{\,*}) \tag{3.31}$$

By rewriting (3.30) and (3.31) in matrix form, one has :

$$\begin{bmatrix} h_A \\ h_B \end{bmatrix} = \begin{bmatrix} \hat{h}_A' \cdot \hat{h}_A^{\,*} & \hat{h}_B' \cdot \hat{h}_A^{\,*} \\ \hat{h}_A' \cdot \hat{h}_B^{\,*} & \hat{h}_B' \cdot \hat{h}_B^{\,*} \end{bmatrix} \begin{bmatrix} h_A' \\ h_B' \end{bmatrix} \tag{3.32}$$

or,

$$\underline{h}(A,B) = [T]\, \underline{h}(A',B') \tag{3.33}$$

where :

$$[T] = \begin{bmatrix} \hat{h}_A' \cdot \hat{h}_A^{\,*} & \hat{h}_B' \cdot \hat{h}_A^{\,*} \\ \hat{h}_A' \cdot \hat{h}_B^{\,*} & \hat{h}_B' \cdot \hat{h}_B^{\,*} \end{bmatrix} \tag{3.34}$$

To determine the matrix [T], the relationship between the two sets of the orthogonal basis vectors should be known. For example, let :

$$\hat{h}_A' = \alpha_1\hat{h}_A + \alpha_2\hat{h}_B \tag{3.35}$$

and

$$\hat{h}_B' = \beta_1\hat{h}_A + \beta_2\hat{h}_B \tag{3.36}$$

These two new unit vectors $\hat{h}_A'$, $\hat{h}_B'$ in the orthogonal basis (A',B') should satisfy the orthogonality relationship of (3.29). For (3.29) to hold, the following must be met

$$|\alpha_1|^2 + |\alpha_2|^2 = 1, \tag{3.37a}$$

$$|\beta_1|^2 + |\beta_2|^2 = 1, \text{ and} \tag{3.37b}$$

$$\alpha_1\beta_1^* + \alpha_2\beta_2^* = 0 \tag{3.37c}$$

By substituting (3.35) and (3.36) into (3.34) and using (3.29), the matrix [T] can be written in the form :

$$[T] = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix} \tag{3.38}$$

This means that there exists an infinite number of unitary matrices [T] given by (3.38), which satisfy the conditions (3.37). For example, if we choose :

$$\beta_1 = -\alpha_2^*, \ \beta_2 = \alpha_1^* \tag{3.39}$$

which satisfies (3.37b & c) provided that (3.37a) is satisfied, Eq.(3.38) yields :

$$[T] = \begin{bmatrix} \alpha_1 & -\alpha_2^* \\ \alpha_2 & \alpha_1^* \end{bmatrix} \tag{3.40}$$

Let α_1 and α_2 be defined by a complex parameter ρ and its complex conjugate ρ^* such that the conditions (3.37) are satisfied. For example, one can write :

$$\alpha_1 = \frac{1}{\sqrt{1+\rho\rho^*}} \text{ and } \alpha_2 = \frac{\rho}{\sqrt{1+\rho\rho^*}}, \text{ so that, } \frac{\alpha_2}{\alpha_1} = \rho \tag{3.41}$$

then :

$$[T] = \frac{1}{\sqrt{1+\rho\rho^*}} \begin{bmatrix} 1 & -\rho^* \\ \rho & 1 \end{bmatrix} \tag{3.42}$$

In case no polarization transformation losses are incured, the matrix [T] in (3.42) is a unitary matrix since it satisfies the condition

$$[T^{-1}] = [T^T]^* \tag{3.43}$$

3.4.2 The scattering matrix in the new basis [(S'(A',B')]:

Using the unitary transformation matrix [T] in terms of the complex parameters ρ and ρ^* given by (3.42), one can obtain the scattering matrix [S'(A',B')] in the new basis (A',B') in terms of the elements of the original scattering matrix [(S(A,B)] and ρ, ρ^*.

We now rewrite (3.33) for the incident and the scattered polarization vectors separately. Thus :

$$\underline{h}^i(A,B) = [T]\ \underline{h}^i(A',B') \tag{3.44}$$

and

$$\underline{h}^s(A,B) = [T^*]\underline{h}^s(A',B') \tag{3.45}$$

We use $[T^*]$ instead of $[T]$ in (3.45) in order to preserve the same sense of polarization with respect to both coordinate systems of the incident and scattered radiation (Graves 1956, Maffett 1968). From (3.45) :

$$\underline{h}^s(A',B') = [T^*]^{-1}\underline{h}^s(A,B) = [T^T]\underline{h}^s(A,B) \tag{3.46}$$

Using (3.21) and (3.44) one has :

$$\begin{aligned}\underline{h}^s(A,B) &= [S(A,B)]\underline{h}^i(A,B)\\ &= [S(A,B)][T]\underline{h}^i(A',B')\end{aligned} \tag{3.47}$$

Substituting (3.47) into (3.46), we obtain :

$$\begin{aligned}\underline{h}^s(A',B') &= [T^T][S(A,B)][T]\underline{h}^i(A',B')\\ &= [S'(A',B')]\underline{h}^i(A',B')\end{aligned} \tag{3.48}$$

where :

$$[S'(A',B')] = [T^T][S(A,B)][T] \tag{3.49}$$

From (3.49) and (3.42), we obtain :

$$\begin{aligned}S'_{A'A'} &= (1+\rho\rho^*)^{-1}[S_{AA}+\rho^2 S_{BB}+\rho(S_{AB}+S_{BA})]\\ S'_{A'B'} &= (1+\rho\rho^*)^{-1}[-\rho^* S_{AA}+\rho S_{BB}+S_{AB}-\rho\rho^* S_{BA}]\\ S'_{B'A'} &= (1+\rho\rho^*)^{-1}[-\rho^* S_{AA}+\rho S_{BB}+S_{BA}-\rho\rho^* S_{AB}]\\ S'_{B'B'} &= (1+\rho\rho^*)^{-1}[\rho^{*2} S_{AA}+S_{BB}-\rho^*(S_{AB}+S_{BA})]\end{aligned} \tag{3.50}$$

Using (3.50) we find that the det{[S(A,B)]} and the span {[S(A,B)]} are transformation invariants (Appendix D), according to :

$$\det\{[S(A,B)]\} = \det\{[S'(A',B')]\} = \text{invariant}, \tag{3.51}$$

and

$$\begin{aligned} \mathrm{span}\{[S(A,B)]\} &= |S_{AA}|^2+|S_{AB}|^2+| S_{BA}|^2+|S_{BB}|^2 = p \\ &= \mathrm{span}\{[S'(A',B')]\} \\ &= |S'_{A'A'}|^2+|S'_{A'B'}|^2+|S'_{B'A'}|^2+|S'_{B'B'}|^2 \\ &= \text{invariant} \end{aligned} \tag{3.52}$$

We note that, if $S_{AB} = S_{BA}$, then $S'_{A'B'} = S'_{B'A'}$, i.e. , if reciprocity is satisfied for any one pair of orthogonal polarizations, it is satisfied for all such pairs. Furthermore, we must emphasize the important property that for any one given aspect and for one frequency, the transformation occurs on one and the same polarization sphere of radius p = span{[S(A,B)]} = span {[S'(A',B')]}. Thus, if [S(A,B)] is known, the new scattering matrix [S'(A',B')] for any other orthogonal base (A',B') can be determined if the relationship between the two sets of bases (A,B) and (A',B') are known. This is shown for example in the transformation from linear to circular polarization basis vectors in (Long 1966). In case of polarization losses, the properties of the coherency matrix need to be used (Kraus 1966), and transformation will not occur on the same polarization sphere (Deschamps 1951), as is discussed in (Thiel 1970) and will be further analyzed in one of our forthcoming reports.

3.5 The Optimal Polarizations :

It was first shown by (Kennaugh 1949-1954), that there exist two pairs of optimal polarization which can be associated with (3.50) and are useful to express the five independent real components of the scattering matrix [S] in the monostatic relative phase case on the polarization sphere of radius p subject to (3.52).

The CO-POLARIZATION (COPOL) NULL PAIRS for minimal polarization are obtained from (3.50) by setting $S'_{A'A'}$ and/or $S'_{B'B'}$ to zero so that in the bistatic case :

$$\rho^{co}_{1,2} = \frac{-(S_{AB}+S_{BA}) \pm \sqrt{(S_{AB}+S_{BA})^2 - 4S_{AA}S_{BB}}}{2S_{BB}} \qquad (3.53)$$

reducing for the monostatic case to :

$$\rho^{co}_{1,2} = \frac{-S_{AB} \pm \sqrt{S_{AB}^2 - S_{AA}S_{BB}}}{S_{BB}} \qquad (3.54)$$

The CROSS-POLARIZATION (XPOL) NULL PAIR for maximal polarization is obtained from (3.50) by setting $S'_{A'B'}$ or $S'_{B'A'}$ to zero so that in the bistatic case :

$$\rho^{x}_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (3.55a)$$

where :

$$a = S_{BB}S_{BA}^{*} + S_{AA}^{*}S_{BA}$$

$$b = -\{|S_{BB}|^2 - S_{AB}S_{BA}^{*} + S_{AB}^{*}S_{BA} - |S_{AA}|^2\}$$

$$c = -(S_{BB}^{*}S_{AB} + S_{AA}S_{AB}^{*}) \qquad (3.55b)$$

reducing for the monostatic case to :

$$b = -\{|S_{BB}|^2 - |S_{AA}|^2\} \text{ and } c = -a^* \tag{3.55c}$$

If we let, in general :

$$u = \frac{1-j\rho}{1+j\rho} \tag{3.56}$$

with ρ being the transformation parameter defined by Eq.(3.50), the coordinates resulting for (3.53) to (3.55) on the Poincare sphere are given by :

$$\text{Colatitude : } \theta = \arccos \frac{|u|^2-1}{|u|^2+1} \tag{3.57}$$

$$\text{Longitude : } \phi' = -\arctan \frac{\mathrm{Im}\{u\}}{\mathrm{Re}\{u\}} = -\mathrm{phase}\{u\} \tag{3.58}$$

On the basis of these expressions, examples of calculations of the optimal polarization pairs for radar targets and clutter are given in Chapter Four. It should be noted, as is illustrated in Fig.(3.14), that for the monostatic case the XPOL and COPOL nulls lie on one main circle, that the XPOL nulls are antipodal, and the connecting line bisects the great circle arc between the COPOL nulls. This means that, if we determine the COPOL nulls, it is easy to find the location of the XPOL nulls but not vice versa. Also, if one XPOL null and one COPOL null are given (e.g. by measurements), the location of the other two nulls can be found. By using the COPOL null polarizations, the scattering

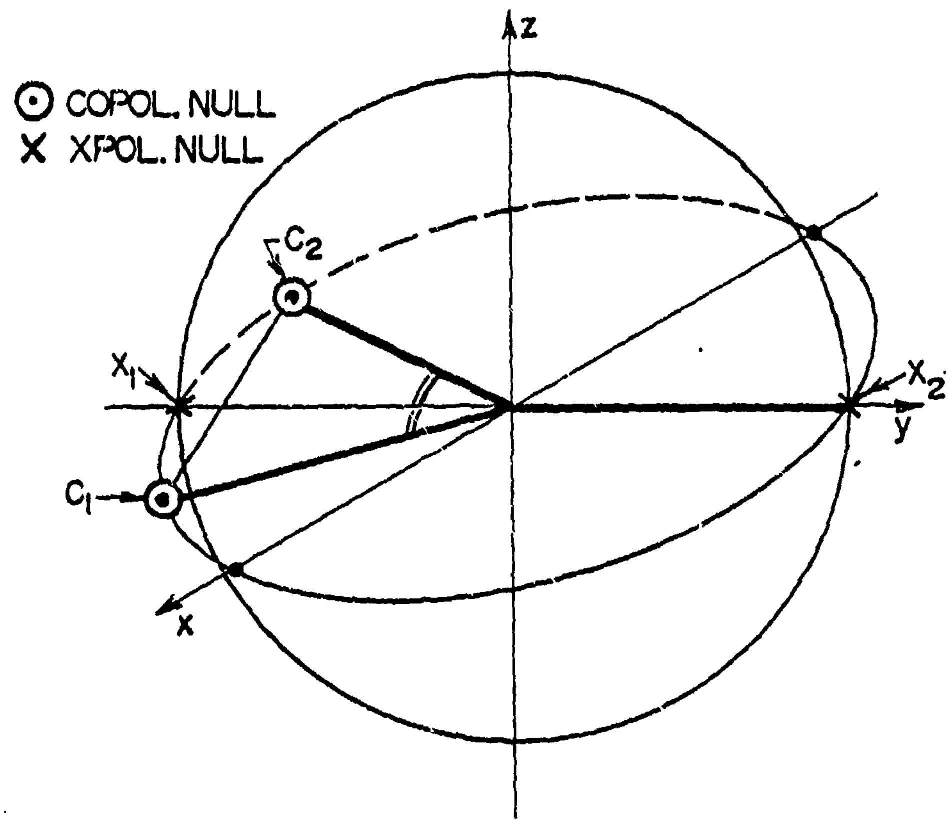


Fig. 3.14 : The Polarization Fork (Huynen 1970).

matrix can be graphically represented by two points on the surface of the Poincare sphere of radius p. The coordinates of the scattering matrix or of the target signature and the radius of the sphere represent the function of the target reflectivity, and in principle contain the same information as [S] for the monostatic relative phase case.

Details of the intrinsic properties of the optimal polarization nulls on the Poincare sphere will be discussed in one of our forthcoming reports together with the analysis of model generated and measurement data.

3.6 The Stokes or Mueller Matrix and its Relationship with the Scattering Matrix :

Whereas, in the coherent case the elements of the 2x2 Sinclair matrix [S(A,B)] are additive, in the incoherent case, the time-averaged Stokes parameters of the 4x4 Mueller matrix are additive. The Mueller matrix [M] relates the scattered and the incident Stokes vectors in the following way :

$$\underline{\underline{g}}^{s}(A,B) = [M]\underline{\underline{g}}^{i}(A,B) \tag{3.59}$$

where the Stokes vector $\underline{\underline{g}} = (g_0, g_1, g_2, g_3) = (I,Q,U,V)$ is given by (3.9) in terms of the phasors of its corresponding polarization vector :

$$\underline{h} = h_A \hat{h}_A + h_B \hat{h}_B \tag{3.60}$$

Similarly, the modified Mueller matrix [Mm] relates the modified scattered and incident Stokes vectors through the equation :

$$\underline{g}_m^{\,s}(A,B) = [Mm]\underline{g}_m^{\,i}(A,B) \tag{3.61}$$

where $\underline{g}_m = (\frac{1}{2}(I+Q), \frac{1}{2}(I-Q), U, V)$ given by (3.11).

In the following sections we will, as given by (3.11), derive the relationship between the matrices [M], [Mm] and [S]. In the first subsection, the derivation of the elements of [M] and [Mm] from [S] is given, while in the second subsection, the inverse problem, i.e., the derivation of the elements of [S] from [M] or [Mm] is solved. Also the relation between [M] and [Mm] is given in the third subsection.

3.6.1 Derivation of the Mueller [M] or Modified Mueller [Mm] Matrices from the Scattering Matrix [S] :

a) Derivation of the Mueller Matrix [M] from the scattering matrix [S] :

The scattering matrix [S(A,B)] is given in the orthogonal basis (A,B) by (3.22) which in the bistatic case has four complex quantities or it possesses eight real quantities for the absolute phase case. The complex quantities are S_{AA}, S_{AB}, S_{BA} and S_{BB} ,while the real quantities are $|S_{AA}|$, $|S_{AB}|$, $|S_{BA}|$, $|S_{BB}|$, ϕ_{AA}, ϕ_{AB}, ϕ_{BA}, ϕ_{BB}. In the monostatic case, we have $S_{BA} = S_{AB}$ or six real quantities $|S_{AA}|$, $|S_{AB}|$, $|S_{BB}|$, ϕ_{AA}, ϕ_{AB}, ϕ_{BB} because $|S_{BA}| = |S_{AB}|$ and $\phi_{BA} = \phi_{AB}$. In this section the elements of the Mueller Matrix [M] for both the bistatic and monostatic cases are derived from the elements of the scattering matrix [S(A,B)], as shown in Appendix (E)

i) Bistatic Case :

$$m_{11} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2+|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2-|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^*+S_{BA}S_{BB}^*\}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^*+S_{BA}S_{BB}^*\}$$

$$m_{21} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2+|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2-|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{23} = \mathrm{Re}\{S_{AA}S_{AB}^*-S_{BA}S_{BB}^*\}$$

$$m_{24} = \mathrm{Im}\{S_{AA}S_{AB}^*-S_{BA}S_{BB}^*\} \qquad (3.62)$$

$$m_{31} = \mathrm{Re}\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{32} = \mathrm{Re}\{S_{AA}S_{BA}^*-S_{AB}S_{BB}^*\}$$

$$m_{33} = \mathrm{Re}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}$$

$$m_{34} = \mathrm{Im}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}$$

$$m_{41} = -\mathrm{Im}\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{42} = -\mathrm{Im}\{S_{AA}S_{BA}^*-S_{AB}S_{BB}^*\}$$

$$m_{43} = -\mathrm{Im}(S_{AA}S_{BB}^*+S_{AB}S_{BA}^*)$$

$$m_{44} = \mathrm{Re}(S_{AA}S_{BB}^*-S_{AB}S_{BA}^*)$$

ii) Monostatic Case :

$$m_{11} = \tfrac{1}{2}\{|S_{AA}|^2+2|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BB}|^2\}$$

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^*+S_{AB}S_{BB}^*\}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^{*}+S_{AB}S_{BB}^{*}\}$$

$$m_{21} = m_{12}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BB}|^2\}-|S_{AB}|^2$$

$$m_{23} = \mathrm{Re}\{S_{AA}S_{AB}^{*}-S_{AB}S_{BB}^{*}\}$$

$$m_{24} = \mathrm{Im}\{S_{AA}S_{AB}^{*}-S_{AB}S_{BB}^{*}\}$$

$$m_{31} = m_{13}$$

$$m_{32} = m_{23}$$

$$m_{33} = \mathrm{Re}\{S_{AA}S_{BB}^{*}\}+|S_{AB}|^2$$

$$m_{34} = \mathrm{Im}\{S_{AA}S_{BB}^{*}\}$$

$$m_{41} = -m_{14}$$

$$m_{42} = -m_{24}$$

$$m_{43} = -m_{34}$$

$$m_{44} = m_{33}+m_{22}-m_{11} \tag{3.63}$$

In this case, we have a maximum of seven independent elements. We note from (3.62) and (3.63) that, all elements of the Mueller matrix [M] are real.

b) <u>Derivation of the modified Mueller matrix [Mm] from the scattering matrix [S]</u> :

Using the definition of [Mm] in Eq.(3.61) and of g_m in Eq.(3.11), the elements of the modified Mueller matrix [Mm] can be obtained directly from the scattering matrix [S(A,B)] (Appendix F); thus for the bistatic case, we obtain

$$[M_m] = \begin{bmatrix} |S_{AA}|^2 & |S_{AB}|^2 & \mathrm{Re}\{S_{AA}S_{AB}^*\} & \mathrm{Im}\{S_{AA}S_{AB}^*\} \\ |S_{BA}|^2 & |S_{BB}|^2 & \mathrm{Re}\{S_{BA}S_{BB}^*\} & \mathrm{Im}\{S_{BA}S_{BB}^*\} \\ 2\mathrm{Re}\{S_{AA}S_{BA}^*\} & 2\mathrm{Re}\{S_{AB}S_{BB}^*\} & \mathrm{Re}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\} & \mathrm{Im}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\} \\ -2\mathrm{Im}\{S_{AA}S_{BA}^*\} & -2\mathrm{Im}\{S_{AB}S_{BB}^*\} & -\mathrm{Im}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\} & \mathrm{Re}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\} \end{bmatrix} \qquad (3.64)$$

and for the monostatic case we only have seven independent elements, since

$M_{21} = M_{12}$, $M_{31} = 2M_{13}$, $M_{41} = -2M_{14}$, $M_{32} = 2M_{23}$,

$M_{42} = -2M_{24}$, $M_{43} = -M_{34}$ and $M_{44} = M_{33}-2M_{12}$

3.6.2 Derivation of the Scattering Matrix [S] from the Mueller [M] or Modified Mueller [Mm] Matrices :

Since it is useful to characterize clutter behaviour by its optimal polarization properties, it is desirable to express the amplitudes and the phases of the scattering matrix elements S_{AA}, S_{AB}, S_{BA} and S_{BB} in terms of the elements of the Mueller matrix elements m_{ij} or of the modified one denoted here by Mij.

a) Derivation of the scattering matrix [S] from the Mueller matrix [M] :

As shown in Appendix (G), for the bistatic case; the amplitudes of the elements of the matrix [S] are given in terms of the elements of [M] by :

$$\begin{aligned} |S_{AA}| &= \sqrt{\tfrac{1}{2}(m_{11}+m_{12}+m_{21}+m_{22})} \\ |S_{AB}| &= \sqrt{\tfrac{1}{2}(m_{11}-m_{12}+m_{21}-m_{22})} \\ |S_{BA}| &= \sqrt{\tfrac{1}{2}(m_{11}+m_{12}-m_{21}-m_{22})} \\ |S_{BB}| &= \sqrt{\tfrac{1}{2}(m_{11}-m_{12}-m_{21}-m_{22})} \end{aligned} \qquad (3.65)$$

and their phases by :

$$\phi_{AA} = \phi_{AB} + \arctan \frac{m_{14}+m_{24}}{m_{13}+m_{23}}$$

ϕ_{AB} is arbitrary (=0)

$$\phi_{BA} = \phi_{AB} + \arctan \frac{m_{14}-m_{24}}{m_{13}-m_{23}} + \arctan \frac{m_{41}-m_{42}}{m_{31}-m_{32}}$$

$$\phi_{BB} = \phi_{AB} + \arctan \frac{m_{41}-m_{42}}{m_{31}-m_{32}} \tag{3.66}$$

which simplify in the monostatic case so that

$|S_{AB}| = |S_{BA}|$ and $\phi_{AB} = \phi_{BA}$.

b) Derivation of the Scattering Matrix [S] from the Modified Mueller Matrix [Mm] :

Similary for the case of the modified Mueller matrix [Mm] as shown in (Appendix H), we obtain for the bistatic case :

$$|S_{AA}| = \sqrt{M_{11}}\ , \ \phi_{AA} = \phi_{AB} + \arctan \frac{M_{14}}{M_{13}}$$

$$|S_{AB}| = \sqrt{M_{12}}\ , \ \phi_{AB} \text{ is arbitrary } (=0)$$

$$|S_{BA}| = \sqrt{M_{21}}\ , \ \phi_{BA} = \phi_{AB} + \arctan \frac{M_{34}+M_{43}}{M_{33}-M_{44}} \tag{3.67}$$

$$|S_{BB}| = \sqrt{M_{22}}\ ,\ \phi_{BB} = \phi_{AB} + \arctan \frac{M_{42}}{M_{32}}$$

which for the monostatic case simplifies so that

$|S_{AB}| = |S_{BA}|$ and $\phi_{AB} = \phi_{BA}$.

<u>3.6.3 The relation between the Mueller matrix [M] and the modified Mueller matrix [Mm]</u> :

For completeness, the relationship between the Mueller [M] and modified Mueller [Mm] matrices is derived in this section. Rewriting (3.59) and (3.61) which defines [M] and [Mm] matrices as follows :

$$\underline{g}^{s}(A,B) = [M]\underline{g}^{i}(A,B) \tag{3.68}$$

$$\underline{g}_m^{s}(A,B) = [Mm]\underline{g}_m^{i}(A,B) \tag{3.69}$$

where $\underline{g} = (I,Q,U,V)$ is the Stokes vector which is defined in Eq.(3.9) and $\underline{g}_m = (\frac{1}{2}(I+Q), \frac{1}{2}(I-Q), U, V)$ is the modified Stokes vector. From the definitions of $\underline{g}$ and $\underline{g}_m$, one can get the relation :

$$\underline{g}_m(A,B) = \begin{bmatrix} \frac{1}{2}(I+Q) \\ \frac{1}{2}(I-Q) \\ U \\ V \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} \tag{3.70}$$

or $$\underline{g}_m(A,B) = [R]\underline{g}(A,B) \tag{3.71}$$

where

$$[R] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.72}$$

From (3.71), we can write :

$$\underline{g}_m^{s}(A,B) = [R]\underline{g}^{s}(A,B) \tag{3.73}$$

From (3.73) and (3.68), then :

$$\underline{g}_m{}^s(A,B) = [R]\ \underline{g}^s(A,B) = [R][M]\ \underline{g}^i(A,B)$$

$$= [R][M][R^{-1}]\underline{g}_m{}^i(A,B) \tag{3.74}$$

where $[R^{-1}]$ is the inverse of the real matrix [R] of (3.72) which can be written as :

$$[R^{-1}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.75}$$

From (3.69) and (3.74), one can write

$$[Mm] = [R][M][R^{-1}] \tag{3.76}$$

or

$$[M] = [R^{-1}][Mm][R] \tag{3.77}$$

In summary, the relation between [M] and [Mm] is derived and is given by the pair of (3.76) and (3.77), where [R] and $[R^{-1}]$ are given by (3.72) and (3.75), respectively.

3.7 Reconstruction of the scattering matrix [S] and the Mueller matrix [M] from the optimal polarizations known on the Poincare sphere :

In this section the reconstruction of the scattering matrix [S] and the Mueller matrices [M] and [Mm]is derived assuming that the COPOL null pair or one COPOL and one XPOL polarization null are given. It will be shown that knowing only the XPOL null pair is not sufficient to reconstruct these matrices as expected. To solve this problem,we have to use the equations in the previous sections of this Chapter. The problem now is: given two polarization nulls, either two COPOL nulls or one COPOL and one XPOL null but not two XPOL nulls, such that their representation on the Poincare sphere are given by (p,θ_1,ϕ_1') and (p,θ_2,ϕ_2'), where p=span{[S]} is the radius of the Poincare sphere, θ is the colatitude and ϕ' is the longitude. Then, it is required to reconstruct the scattering matrix [S] with relative phase and similarly the Mueller matrices [M] and [Mm]. First, we have to calculate the auxiliary parameter ρ which is mentioned in Section (3.5) from knowing θ, ϕ' at any polarization null. Using (3.56), one can get

$$\rho = -j\,\frac{1-u}{1+u} \tag{3.78}$$

where u is a complex number and it can be calculated using (3.57) and (3.58) as follows :

$$u = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\; e^{-j\phi'} \tag{3.79}$$

where (θ,ϕ') is the coordinate of one of the polarization null on the Poincare sphere. This means if (θ,ϕ') is known for such a polarization null, its corresponding complex parameter ρ can be calculated using (3.78) and (3.79). Next, we will calculate the scattering and the Mueller matrices from the optimal polarizations.

<u>3.7.1 the COPOL null pair is known</u> .

Let the COPOL polarization null pair $(p,\theta_1^{co},\phi_1^{co})$ and $(p,\theta_2^{co},\phi_2^{co})$ be given, where we know from Section 3.5 that p=span{[S]}. By using

(3.78) and (3.79) we can calculate the corresponding complex parameters ρ_1^{co}, ρ_2^{co}. Each of these parameters should be a root of the first equation of (3.50). This implies for the monostatic case :

$$S_{AA}+(\rho_1^{co})^2 S_{BB}+2\rho_1^{co} S_{AB} = 0 \tag{3.80}$$

and

$$S_{AA}+(\rho_2^{co})^2 S_{BB}+2\rho_2^{co} S_{AB} = 0 \tag{3.81}$$

where S_{AA}, S_{AB} and S_{BB} are the elements of the scattering matrix [S(A,B)] in the orthogonal basis (A,B). From (3.80) and (3.81), one can calculate two elements of the scattering matrix in terms of the third one. For example, solving both equations to calculate S_{AA}, S_{BB} in torms of S_{AB}, one can write :

$$S_{AA} = \frac{-2\rho_1^{co}\rho_2^{co}}{\rho_1^{co}+\rho_2^{co}} S_{AB} \tag{3.82}$$

and

$$S_{BB} = \frac{-2}{\rho_1^{co}+\rho_2^{co}} S_{AB} \tag{3.83}$$

then the scattering matrix can be written in the form :

$$[S(A,B)] = S_{AB} \begin{bmatrix} -\dfrac{2\rho_1^{co}\rho_2^{co}}{\rho_1^{co}+\rho_2^{co}} & 1 \\ 1 & \dfrac{-2}{\rho_1^{co}+\rho_2^{co}} \end{bmatrix} \tag{3.84}$$

Now we have to calculate S_{AB}, where by using (3.52)

$$|S_{AA}|^2+2|S_{AB}|^2+|S_{BB}|^2 = p = \mathrm{span}\{[S(A,B)]\} \tag{3.85}$$

We substitute (3.82) and (3.83) into (3.85), then

$$|S_{AB}| = \frac{\sqrt{p}}{\sqrt{2}} \frac{|\rho_1^{co}+\rho_2^{co}|}{\sqrt{|\rho_1^{co}+\rho_2^{co}|^2+2|\rho_1^{co}\rho_2^{co}|^2+2}} \tag{3.86}$$

The absolute phase of S_{AB} cannot be reconstructed from the knowledge of the optimal polarization pairs as was clearly shown by Kennaugh(1952). The scattering matrix with relative phase only can be reconstructed. Letting $\phi_{AB}=0$ then $S_{AB}=|S_{AB}|$. Substituting from (3.86) into (3.84), one can write the reconstructed scattering matrix [S] with relative phase in terms of the COPOL null pair $(\rho_1^{co},\rho_2^{co})$ as follows :

$$[S(A,B)] = K \begin{bmatrix} a & b \\ b & c \end{bmatrix} \tag{3.87}$$

where

$K=\sqrt{p/2}\{|\rho_1^{co}+\rho_2^{co}|^2+2|\rho_1^{co}\rho_2^{co}|^2+2\}^{-\frac{1}{2}}$,

$a = -2\rho_1^{co}\rho_2^{co}\exp(-j\phi_E)$,

$b = |\rho_1^{co}+\rho_2^{co}|$,

$c = -2\exp(-j\phi_E)$ and

$\phi_E = \mathrm{phase}(\rho_1^{co}+\rho_2^{co})$

3.7.2 One COPOL and one XPOL null are known :

Let (ρ_i^{co},ρ_j^{x}) be known on the Poincare sphere of radius p, where ρ_i^{co} is any COPOL null, i=1 or 2 and ρ_j^{x} is any XPOL null, j=1 or 2. ρ_i^{co} should satisfy the first equation of (3.50) and ρ_j^{x} satisfy the second one. This means :

$$S_{AA}+(\rho_i^{co})^2 S_{BB}+2\rho_i^{co}\, S_{AB} = 0 \tag{3.88}$$

and

$$-(\rho_j^{x})^{*} S_{AA}+\rho_j^{x} S_{BB}+S_{AB}(1-|\rho_j^{x}|^2) = 0 \tag{3.89}$$

mulitply (3.88) by $(\rho_j^{x})^{*}$ and add it to (3.89), then :

$$S_{BB} = -S_{AB}\,\frac{2\rho_i^{co}(\rho_j^{x})^{*}-|\rho_j^{x}|^2+1}{(\rho_j^{x})^{*}(\rho_i^{co})^2+\rho_j^{x}} \tag{3.90}$$

Substitute (3.90) into (3.88), then :

$$S_{AA} = S_{AB}\,\frac{\rho_i^{co}-\rho_i^{co}|\rho_j^{x}|^2-2\rho_j^{x}}{(\rho_j^{x})^{*}(\rho_i^{co})^2+\rho_j^{x}}\,\rho_i^{co} \tag{3.91}$$

Using (3.90) and (3.91) and (3.85), then we can write :

$$|S_{AB}| = p/D\,\{|(\rho_j^{x})^{*}(\rho_i^{co})^2+\rho_j^{x}|\} \tag{3.92}$$

where

$$\begin{aligned} D = \{2|(\rho_j^{x})^{*}(\rho_i^{co})^2+\rho_j^{x}|^2 \\ +|\rho_i^{co}|^2|[\rho_i^{co}-\rho_i^{co}|\rho_j^{x}|^2-2\rho_j^{x}]|^2 \\ +|[2\rho_i^{co}(\rho_j^{x})^{*}-|\rho_j^{x}|^2+1]|^2\} \end{aligned} \tag{3.93}$$

From (3.90), (3.91) and (3.92), we can write the scattering matrix with relative phase (ϕ_{AB}=0) as following :

$$[S(A,B)] = \sqrt{\frac{p}{D}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{3.94}$$

where

D is given by (3.92),

$$a = \rho_i^{co}[\rho_i^{co} - \rho_i^{co}|\rho_j^{x}|^2 - 2\rho_j^{x}]e^{-j\phi_E}$$

$$b = |(\rho_j^{x})^*(\rho_i^{co})^2 + \rho_j^{x}|$$

$$c = |(\rho_j^{x})^*(\rho_i^{co})^2 + \rho_j^{x}|$$

$$d = -[2\rho_i^{co}(\rho_j^{x})^* - |\rho_j^{x}|^2 + 1]e^{-j\phi_E}$$

$$\phi_E = \text{phase of } [(\rho_j^{x})^*(\rho_i^{co})^2 + \rho_j^{x}] \tag{3.95}$$

3.7.3 The XPOL null pair is known :

Let (ρ_1^{x}, ρ_2^{x}) be the XPOL null pair, then each value should satisfy the second equation in (3.50), where

$$-(\rho_1^{x})^* S_{AA} + \rho_1^{x} S_{BB} + S_{AB}[1 - \rho_1^{x}(\rho_1^{x})^*] = 0 \tag{3.96}$$

and

$$-(\rho_2^{x})^* S_{AA} + \rho_2^{x} S_{BB} + S_{AB}[1 - \rho_2^{x}(\rho_2^{x})^*] = 0 \tag{3.97}$$

Multiply (3.97) by ρ_1^{x} and subtract the result from (3.96) after multiplication by ρ_2^{x}, then :

$$[-\rho_2^{x}(\rho_1^{x})^* + \rho_1^{x}(\rho_2^{x})^*]S_{AA}$$

$$+S_{AB}[-\rho_1^{x} + \rho_2^{x}(\rho_2^{x})^*\rho_1^{x} + \rho_2^{x} - \rho_1^{x}\rho_2^{x}(\rho_1^{x})^*] = 0 \tag{3.98}$$

Using the relation $\rho_1^{x}(\rho_2^{x})^* = (\rho_1^{x})^*\rho_2^{x} = -1$

i.e. the property of the XPOL nulls to be antipodal, which can be derived easily from (3.55a) and (3.55c). Eq.(3.98) will vanish for any value of ρ_1^x and ρ_2^x. This is also true when one tries to calculate S_{BB} in term of S_{AB}. This means the scattering matrix [S] cannot be determined by using the two XPOL nulls as was previously established by Kennaugh and shown also in Huynen (1970).

In summary, the scattering matrix [S] with relative phase at fixed aspect and at a given frequency can be reconstructed by using (3.87) for the case in which the two COPOL nulls are known, and by using (3.94) for the case in which one COPOL and one XPOL null are known. We note here that [S] cannot be reconstructed if only the two XPOL nulls are known. After calculating the scattering matrix [S], one can calculate the Mueller [M] and the modified Mueller [Mm] matrices in the monostatic case using (3.63) and (3.64), respectively.

CHAPTER FOUR

NUMERICAL RESULTS

4.1 Introduction :

In this Chapter, we will use the theory of Chapter Three to calculate the two unknown matrices given either the scattering [S], the Mueller [M], or the modified Mueller [Mm] matrices for different targets and for sea clutter. We will concentrate on the monostatic relative phase case for the [S] matrix. It should be noted that, the bistatic case also can be calculated by using the formulas of the same Chapter. Also, the COPOL and XPOL nulls, at a given aspect and fixed frequency, and their representation on the Poincare sphere for each case are calculated. It should be noted that, for calculating the scattering matrix [S] from the Mueller matrices [M] or [Mm] in the monostatic relative phase case, only seven elements from [M] or [Mm] are needed, e.g. m_{11}, m_{12}, m_{13}, m_{14}, m_{22}, m_{23} and m_{24} as was shown in (3.63) and (3.64).

4.2 Targets with Simple Shapes : (Huynen 1970)

Example(1) : Large metallic sphere or a flat plate :

In this example a large ideally conducting sphere or flat plate at normal incidence is considered. The target shape is shown in Fig.4.1. The scattering matrix is given by :

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

the Mueller matrices are given by :

$$[M] = [Mm] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Optimal polarization : (Fig.4.2)

a. COPOL nulls :

$\rho_1^{co} = j$, $\theta_1^{co} = 0°$, ϕ_1^{co} arb.

$\rho_2^{co} = -j$, $\theta_2^{co} = 180°$, ϕ_2^{co} arb.

This means the COPOL nulls lie at the North and South poles.

b. XPOL nulls :

They exist anywhere at antipodal locations on the Equator (θ=90° major circle).

In this example, if the polarization of the incident wave is left circular then the return signal will be right circular and vice versa. This means if both the radar transmitting and receiving antennas are adjusted to use left(or right) circular polarization, then the receiver will receive no (or minimum) return signal from the target. Also, the received return signal will be maximum if both antennas are using the same linear polarization.

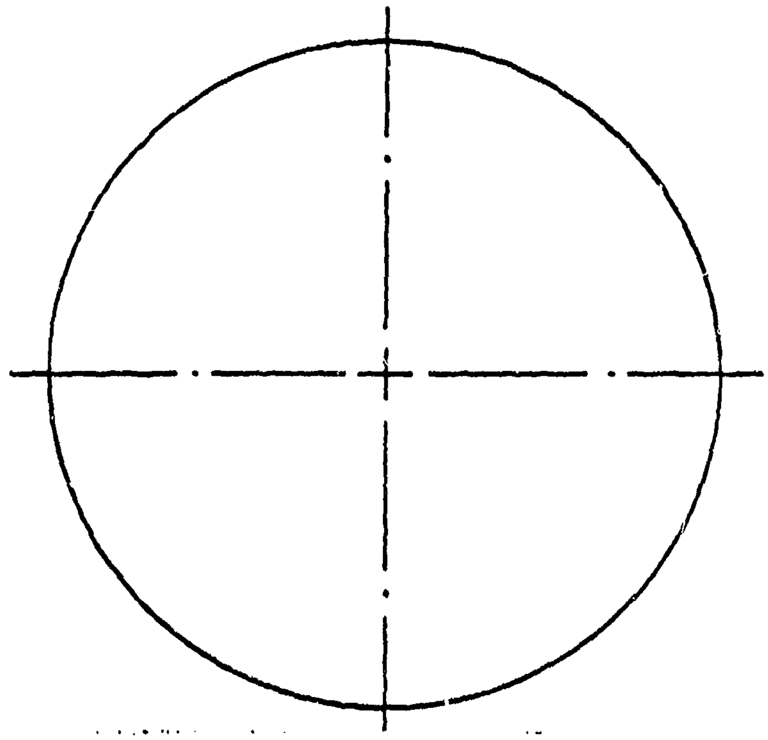


Fig. 4.1 : Spherical Target.

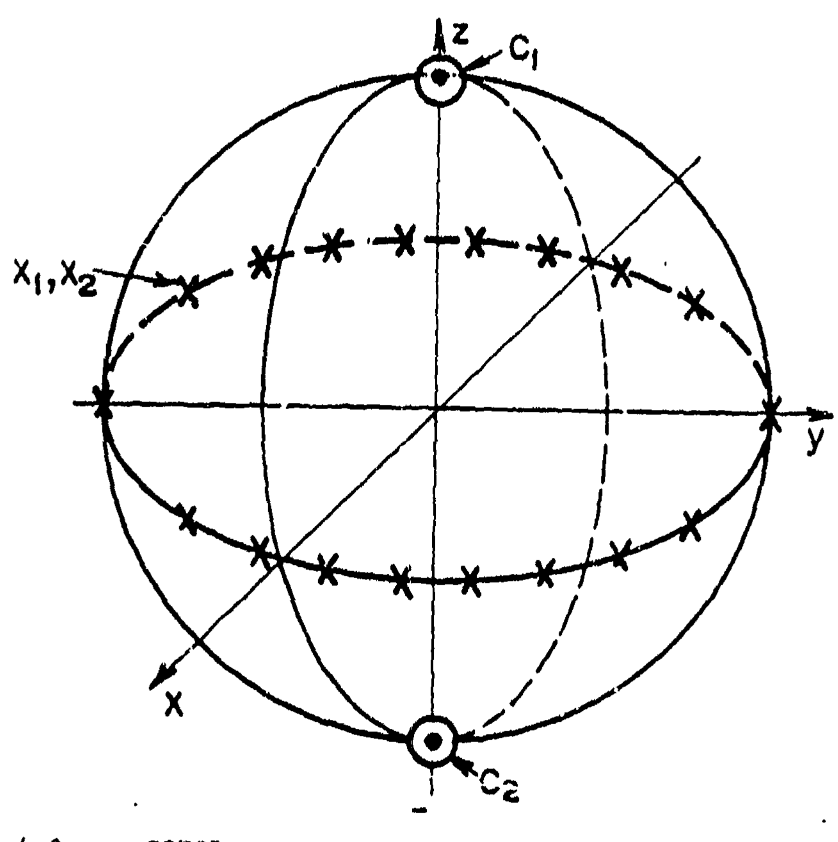


Fig. 4.2 : COPOL and XPOL nulls for a metallic sphere or flat plate.

Example 2: Metallic trough: (Fig.4.3)

The target in this case is a large metallic trough (two planes intersecting at 90°) oriented with axis (the plane's line of intersection) horizontal or vertical. The view angle is considered normal to the trough's open surface. This target has a two-bounce reflection characteristic. The scattering matrix is given by (Huynen 1970) :

$$[S] = \pm\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and the Mueller matrices are :

$$[M] = [Mm] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Optimal polarizations: (Fig. 4.4)

a. COPOL nulls :

$$\rho_1^{co} = -1, \qquad \theta_1^{co} = 90°, \qquad \phi_1^{co} = -90°$$

$$\rho_2^{co} = 1, \qquad \theta_2^{co} = 90°, \qquad \phi_2^{co} = 90°$$

b. XPOL nulls :

They exist anywhere at antipodal locations on the major circle $\phi = 0°$.

NOTE : In comparision with the sphere results of Example 1, we note the important difference in phase of element S_{BB} for Example 2; i.e.; the relative phase relating the two co-polarized elements is of paramount importance.

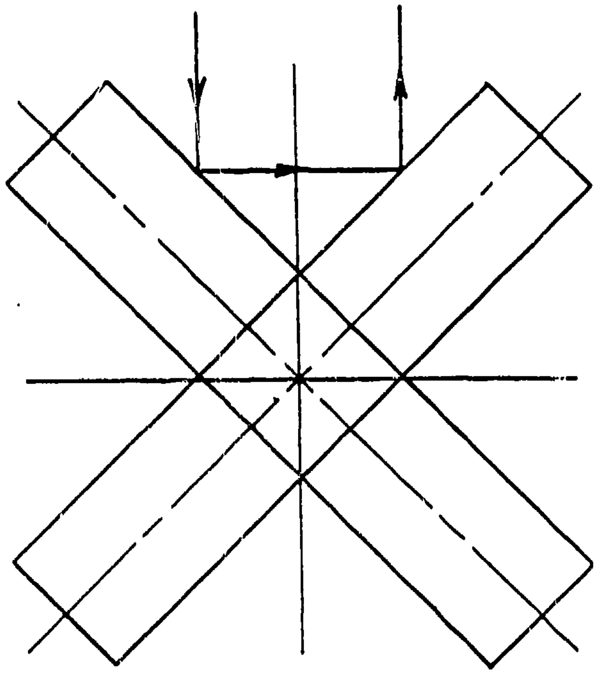


Fig. 4.3 : A metallic Trough (Huynen 1970).

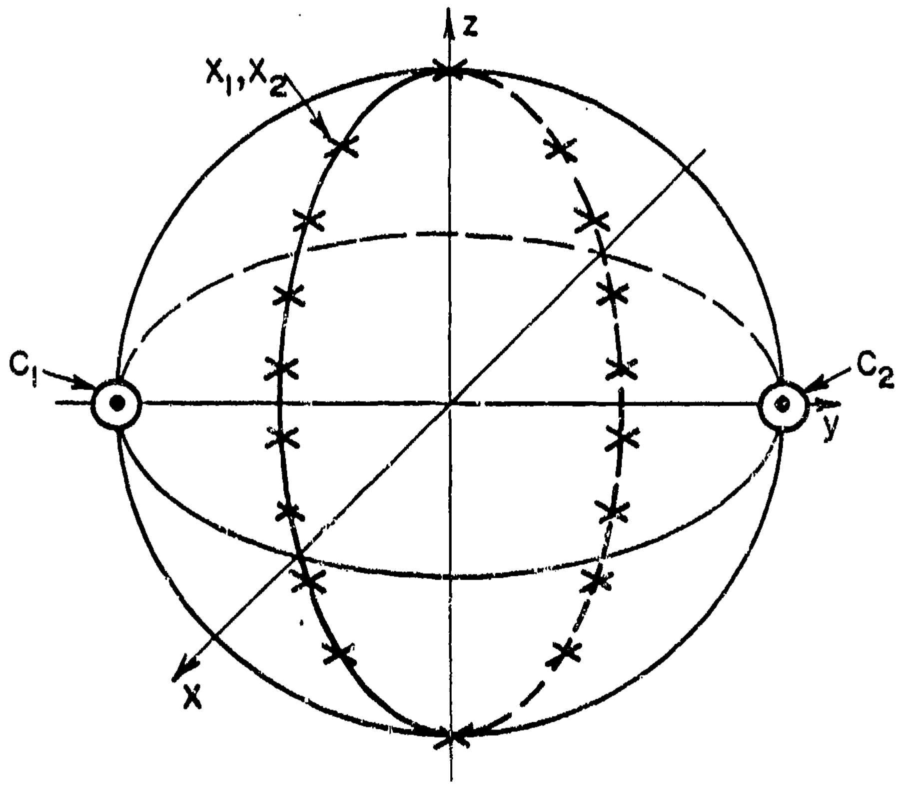


Fig. 4.4 : COPOL and XPOL nulls for a metallic trough.

Example 3 : A metallic helix with right screw. (Fig.4.5)

The scattering matrix in this case is given by (Huynen 1970) :

$$[S] = \frac{1}{2}\begin{bmatrix} 1 & -j \\ -j & -1 \end{bmatrix}$$

the Mueller matrix is :

$$[M] = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}$$

and the modified Mueller matrix is given by :

$$[Mm] = \frac{1}{2}\begin{bmatrix} 0.5 & 0.5 & 0 & 1 \\ 0.5 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & -1 \end{bmatrix}$$

Optimal polarizations : (Fig.4.6)

a) COPOL nulls

$\rho_1^{co} = -j$, $\theta_1^{co} = 180°$, ϕ_1^{co} arb,

$\rho_2^{co} = -j$, $\theta_2^{co} = 180°$, ϕ_2^{co} arb,

The two COPOL nulls lie on the nadir.

b) XPOL nulls :

$\rho_1^{x} = j$, $\theta_1^{x} = 0°$, ϕ_1^{x} arb,

$\rho_2^{x} = -j$, $\theta_2^{x} = 180°$, ϕ_2^{x} arb,

one XPOL null lies on the zenith and the other on the nadir.

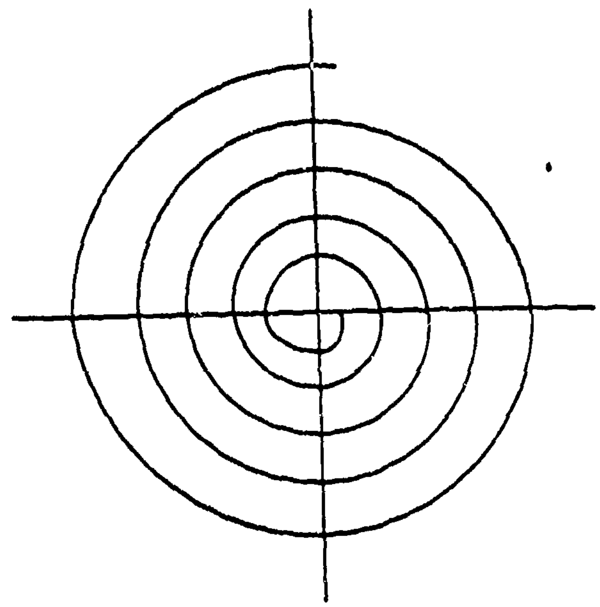


Fig. 4.5 : A metallic helix with right screw (Huynen 1970).

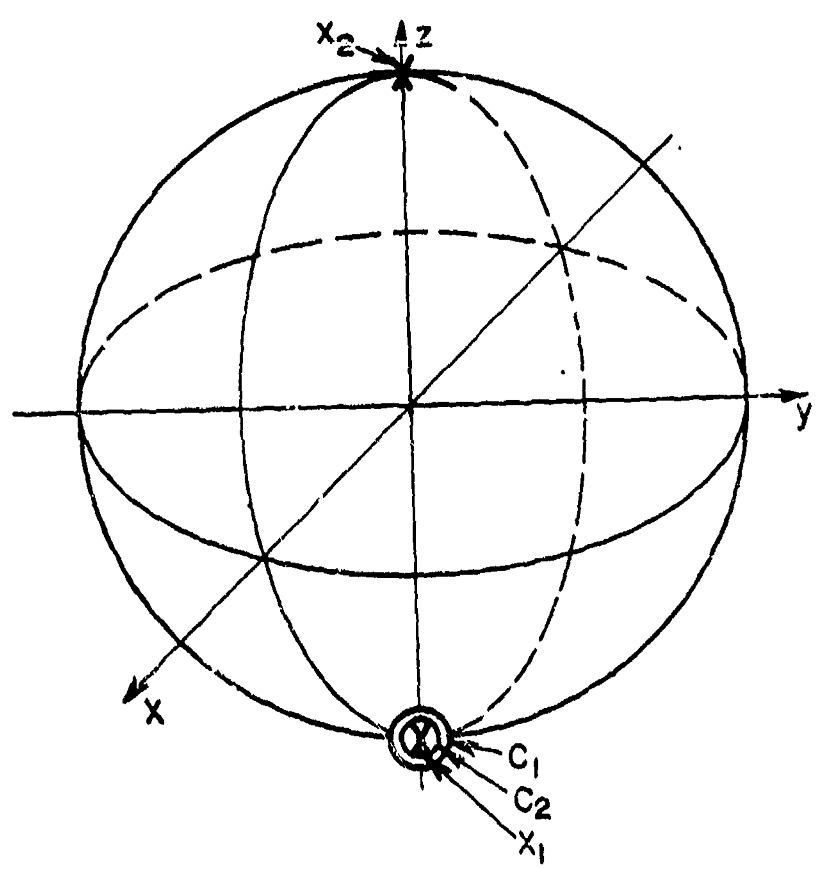


Fig. 4.6 : COPOL and XPOL nulls for a right screw metallic helix.

Example 4 : A metallic helix with left screw. (Fig.4.7)

The scattering matrix for a metallic helix with left screw is (Huynen 1970) :

$$[S] = \frac{1}{2}\begin{bmatrix} 1 & j \\ j & -1 \end{bmatrix}$$

the Mueller matrix is given by :

$$[M] = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

and the modified Mueller matrix is :

$$[Mm] = \frac{1}{2}\begin{bmatrix} 0.5 & 0.5 & 0 & -0.5 \\ 0.5 & 0.5 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Optimal polarizations (Fig.4.8) :

a) COPOL nulls :

$\rho_1^{co} = j, \qquad \theta_1^{co} = 0°, \qquad \phi_1^{co}$ arb,

$\rho_2^{co} = j, \qquad \theta_2^{co} = 0°, \qquad \phi_2^{co}$ arb,

The two COPOL nulls lie at the zenith.

b) XPOL nulls :

$\rho_1^{x} = -j, \qquad \theta_1^{x} = 180° \qquad \phi_1^{x}$ arb.

$\rho_2^{x} = j, \qquad \theta_2^{x} = 0°, \qquad \phi_2^{x}$ arb,

One XPOL null lies on the zenith while the other on the nadir.

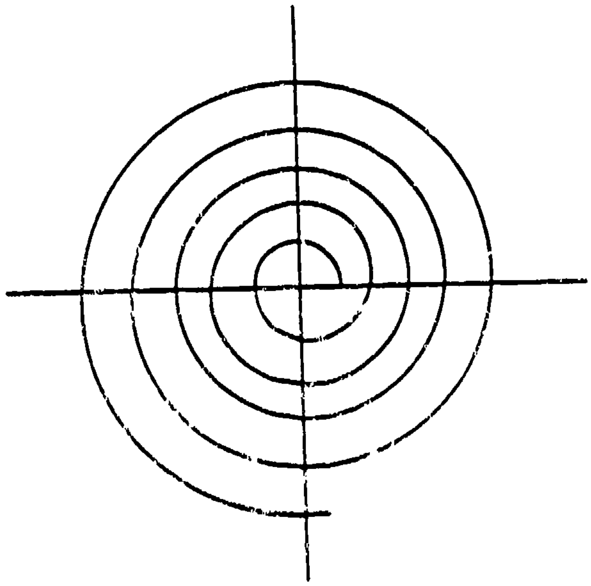


Fig. 4.7 : A metallic helix with left screw (Huynen 1970).

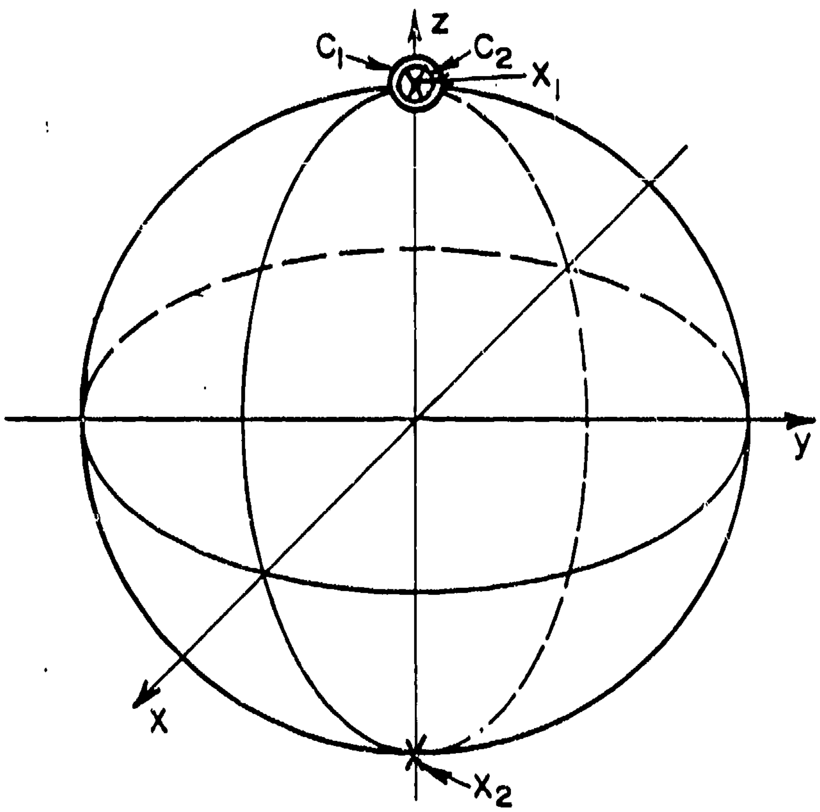


Fig. 4.8 : COPOL and XPOL nulls for a metallic left screw.

Example 5 : Linear target oriented by an angle ψ to the horizontal (x-axis) (Huynen 1970) :

The target model is shown in Fig.4.9. The scattering matrix is given in terms of the angle ψ by :

$$[S(\psi)] = \begin{bmatrix} \cos^2\psi & \sin\psi\cos\psi \\ \sin\psi\cos\psi & \sin^2\psi \end{bmatrix}$$

The Mueller matrices are calculated using (3.63) and (3.64) by :

$$[M(\psi)] = \tfrac{1}{2}\begin{bmatrix} 1 & \cos 2\psi & \sin 2\psi & 0 \\ \cos 2\psi & \cos^2 2\psi & \tfrac{1}{2}\sin 4\psi & 0 \\ \sin 2\psi & \tfrac{1}{2}\sin 4\psi & \sin^2 2\psi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[Mm(\psi)] = \begin{bmatrix} \cos^4\psi & \tfrac{1}{4}\sin^2 2\psi & \cos^3\psi\sin\psi & 0 \\ \tfrac{1}{4}\sin^2 2\psi & \sin^4\psi & \sin^3\psi\cos\psi & 0 \\ 2\cos^3\psi\sin\psi & 2\sin^3\psi\cos\psi & \tfrac{1}{2}\sin^2 2\psi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Optimal polarizations :

a) COPOL nulls :

$$\rho_1^{co}=\rho_2^{co}=-\cot\psi, \quad \theta_1^{co}=\theta_2^{co}=90°, \quad \phi_1^{co}=\phi_2^{co}=2\psi\pm\pi$$

b) XPOL nulls :

$$\rho_1^{x}=\tan\psi, \ \rho_2^{x}=-\cot\psi, \quad \theta_1^{x}=\theta_2^{x}=90°, \quad \phi_1^{x}=\phi_2^{x}=2\psi$$

Special cases :

(i) The target is aligned horizontally ($\psi=0$) :

$$[S(\psi=0)] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rho_1^{co}=\rho_2^{co}=-\infty, \quad \theta_1^{co}=\theta_2^{co}=90°, \quad \phi_1^{co}=\phi_2^{co}=\pi$$

$$\rho_1^{x}=0, \ \rho_2^{x}=-\infty, \quad \theta_1^{x}=\theta_2^{x}=90°, \quad \phi_1^{x}=\phi_2^{x}=0$$

The Co-Pol and X-Pol nulls are shown in Fig.4.10.

(ii) The target is aligned vertically ($\psi=90°$) :

$$[S(\psi=90°)] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\rho_1^{co}=\rho_2^{co}=0,\qquad \theta_1^{co}=\theta_2^{co}=90°,\qquad \phi_1^{co}=\phi_2^{co}=0$

$\rho_1^{x}=\infty,\quad \rho_2^{x}=0,\qquad \theta_1^{x}=\theta_2^{x}=90°,\qquad \phi_1^{x}=\phi_2^{x}=180°$

The COPOL and XPOL nulls are shown in Fig.4.11.

(iii) linear target with $\psi=45°$:

$$[S(\psi=45°)] = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$\rho_1^{co}=\rho_2^{co}=-1,\qquad \theta_1^{co}=\theta_2^{co}=90°,\qquad \phi_1^{co}=\phi_2^{co}=270°$

$\rho_1^{x}=\rho_2^{x}=1,\qquad \theta_1^{x}=\theta_2^{x}=90°,\qquad \phi_1^{x}=\phi_2^{x}=90°$

The COPOL and XPOL nulls are shown in Fig.4.12.

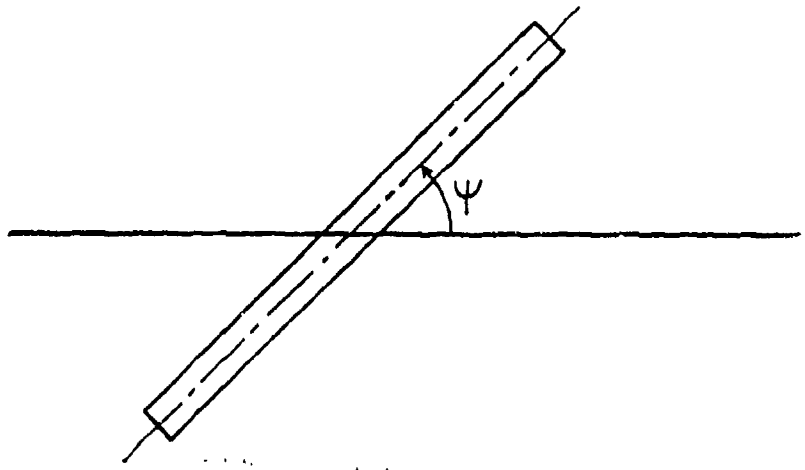


Fig. 4.9 : Linear Target .

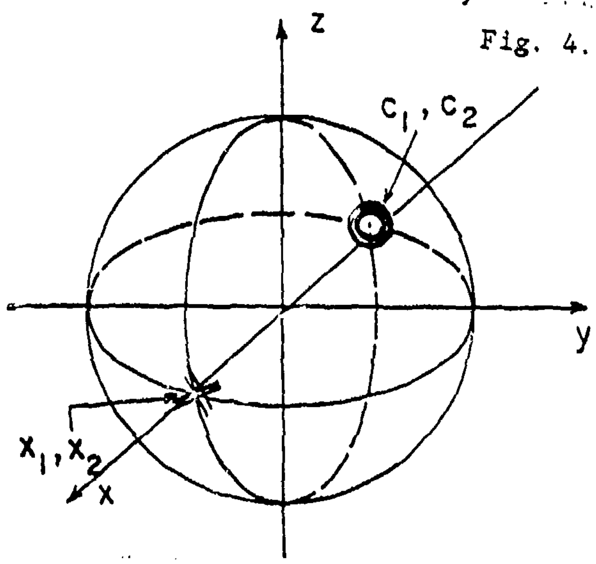


Fig. 4.10 : COPOL and XPOL nulls for a horizontal linear target.

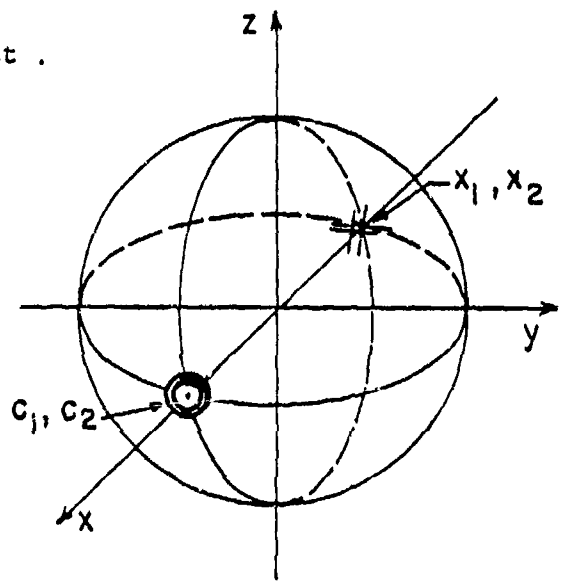


Fig. 4.11 : COPOL and XPOL nulls for a vertical linear target.

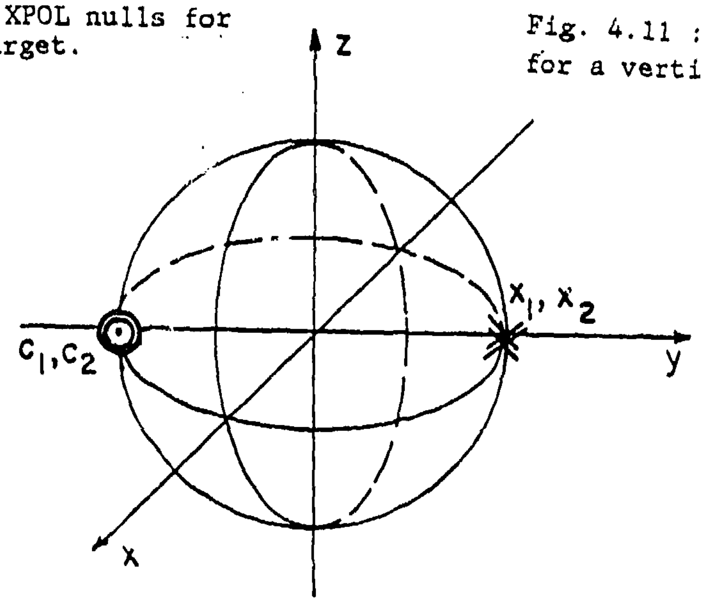


Fig. 4.12 : COPOL and XPOL nulls for a linear target with Ψ=45 .

4.3 Targets for more Complex Shapes : (Crispin et al 1961)

Three targets for more complex shapes are taken into consideration in this section. The data are taken from (Crispin et al 1961) for three target models : missile with and without fins, and the nose cone model. The measured data for the radar cross-section and relative phases are shown in Table(4-1). The radar cross-section were measured relative to a conducting sphere. The geometry for the cross-section study used in The University of Michigan, Radiation Lab. Report (Crispin 1961) is shown in Fig.4.13.

Target	Aspect	RCS in DBS Relative to Sphere			Phases in degrees Rel. to $\phi_{xy}(=0)$		Sphere Diameter
		σ_{xx}	σ_{xy}	σ_{yy}	ϕ_{xx}	ϕ_{yy}	
Nose Cone	26°	3.6	-16.1	2.8	128.0°	121.0°	2.945"
Missile with Fins	30°	-9.3	-20.5	-0.8	98.60°	159.3°	1.98"
Missile without Fins	180°	5.9	-30.0	6.3	34.5°	35.0°	1.98"

Table 4·1: Experimental data for RCS (Crispin et al, 1961)

The frequency is 9.7 GHz and the range R=36½ft. for the nose cone target model and R=33ft. for the missile model.

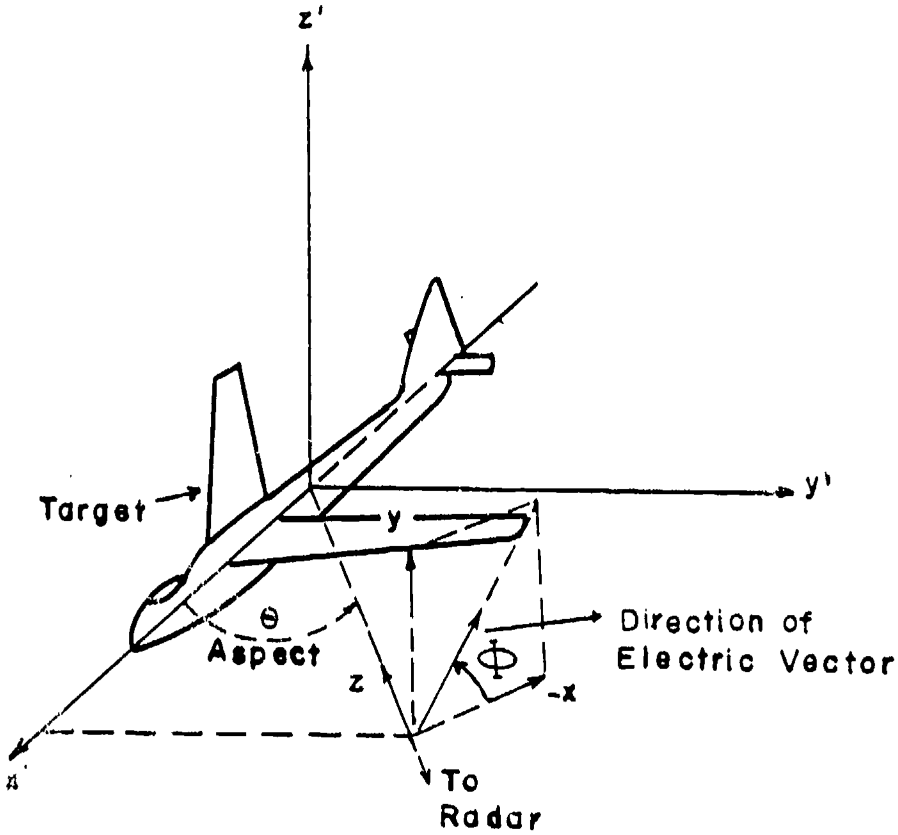


Fig. 4.13 : Geometry for cross section study (Crispin et al 1961).

Example (6): Nose cone model

The geometry of the model is shown in Fig.4.14 and the scattering matrix relative to the sphere with diameter 2.945 inches at aspect angle 26° is given by :

$$[S] = \begin{bmatrix} -0.452+j0.579 & 0.760\times10^{-1} \\ 0.760\times10^{-1} & -0.345+j0.574 \end{bmatrix}$$

the Mueller matrix is :

$$[M] = \begin{bmatrix} 0.5 & 0.454\times10^{-1} & -0.606\times10^{-1} & 0.35\times10^{-3} \\ 0.454\times10^{-1} & 0.488 & -0.815\times10^{-2} & 0.877\times10^{-1} \\ -0.606\times10^{-1} & -0.815\times10^{-2} & 0.494 & 0.6\times10^{-1} \\ -0.35\times10^{-3} & -0.877\times10^{-1} & -0.6\times10^{-1} & 0.483 \end{bmatrix}$$

the modified Mueller matrix is :

$$[Mm] = \begin{bmatrix} 0.54 & 0.578\times10^{-2} & -0.344\times10^{-1} & 0.44\times10^{-1} \\ 0.578\times10^{-2} & 0.449 & -0.262\times10^{-1} & -0.437\times10^{-1} \\ -0.688\times10^{-1} & -0.525\times10^{-1} & 0.494 & 0.6\times10^{-1} \\ -0.88\times10^{-1} & 0.873\times10^{-1} & -0.6\times10^{-1} & 0.483 \end{bmatrix}$$

Optimal polarizations : (Fig.4.15)

a) COPOL nulls :

$\rho_1^{co}=0.1168-j0.9504$, $\theta_1^{co}=172.57°$, $\phi_1^{co}=70.438°$

$\rho_2^{co}=0.1159\times10^{-3}+j1.1450$, $\theta_2^{co}=7.736°$ $\phi_2^{co}=179.957°$

b) XPOL nulls :

$\rho_1^{x}=0.5005+j0.2889\times10^{-2}$, $\theta_1^{x}=89.735°$, $\phi_1^{x}=-53.177°$

$\rho_2^{x}=1.9979-j0.1153\times10^{-1}$, $\theta_2^{x}=90.265°$, $\phi_2^{x}=126.822°$

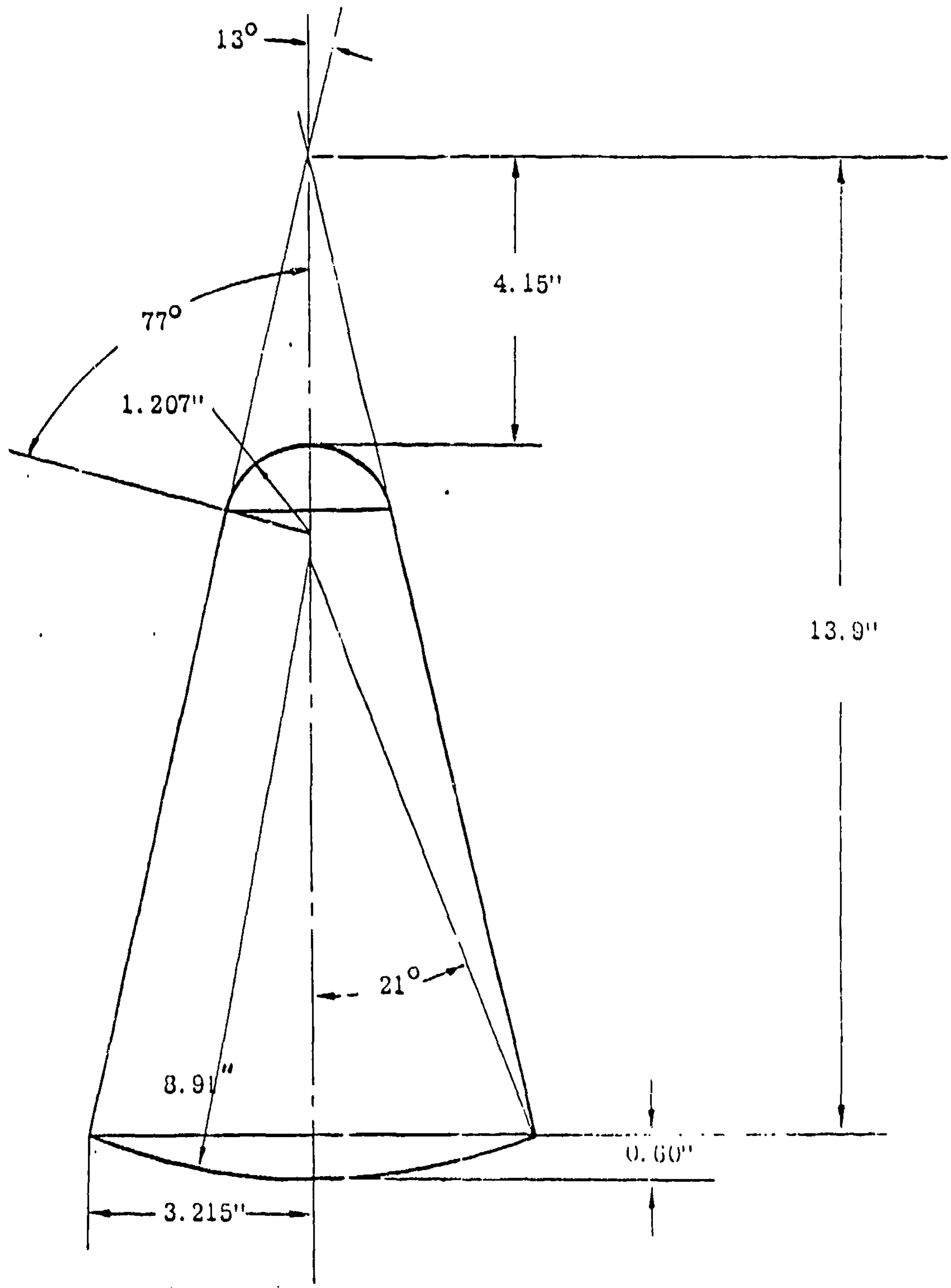


Fig. 4.14 : Nose Cone Model (Crispin et al 1961) .

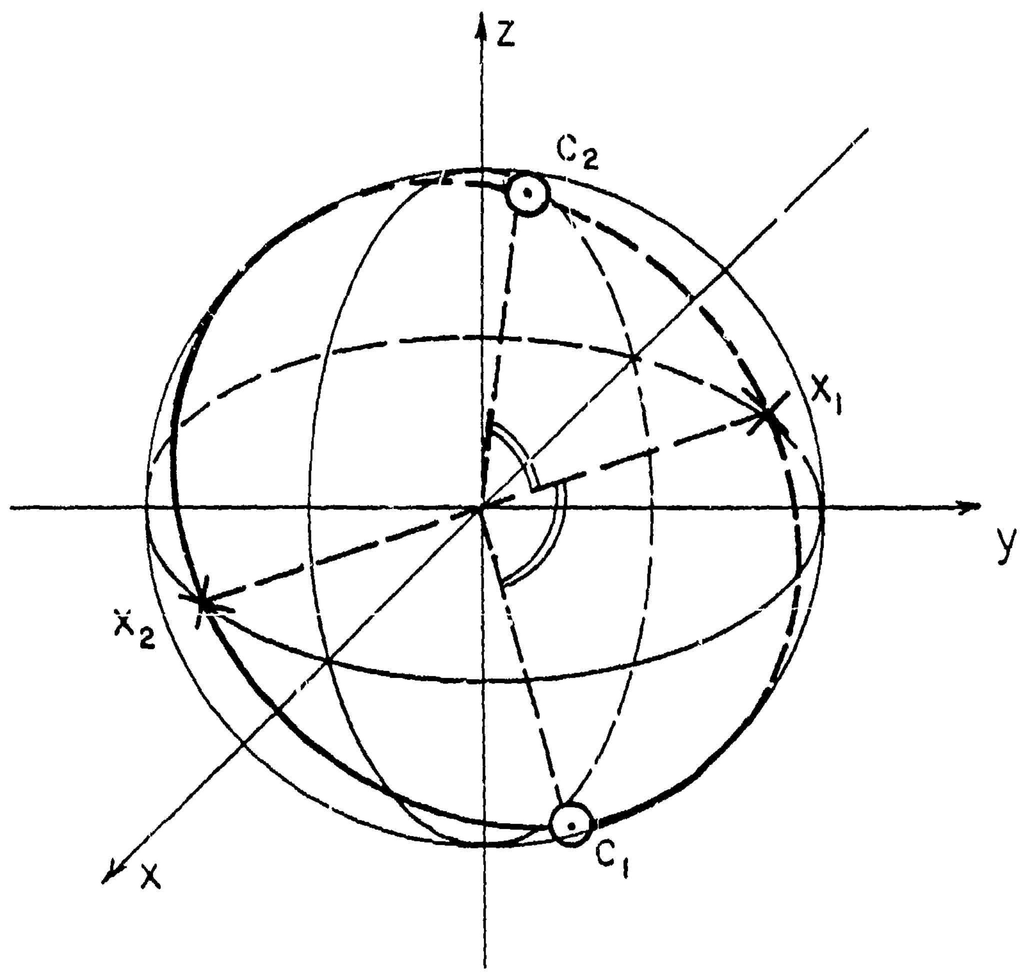


Fig. 4.15 : COPOL and XPOL nulls for a nose cone model.

Example (7): Missile model with fins :

The geometry of this model is shown in Fig.4.16. The scattering matrix is calculated from Table 4.1 at aspect angle 30°. The calculated scattering matrix is given by (relative to a sphere with diameter 1.98") :

$$[S] = \begin{bmatrix} -0.521\text{x}10^{-1}+j0.345 & 0.96\text{x}10^{-1} \\ 0.96\text{x}10^{-1} & -0.868+j0.328 \end{bmatrix}$$

also, the Mueller matrices are given by :

$$[M] = \begin{bmatrix} 0.5 & -0.369 & -0.883\text{x}10^{-1} & 0.161\text{x}10^{-2} \\ -0.369 & 0.482 & 0.783\text{x}10^{-1} & 0.646\text{x}10^{-1} \\ -0.883\text{x}10^{-1} & 0.783\text{x}10^{-1} & 0.167 & -0.282 \\ -0.161\text{x}10^{-2} & -0.646\text{x}10^{-1} & 0.282 & 0.149 \end{bmatrix}$$

$$[Mm] = \begin{bmatrix} 0.121 & 0.922\text{x}10^{-2} & -0.5\text{x}10^{-2} & 0.331\text{x}10^{-1} \\ 0.922\text{x}10^{-2} & 0.86 & -0.833\text{x}10^{-1} & -0.315\text{x}10^{-1} \\ -0.1\text{x}10^{-1} & -0.167 & 0.167 & -0.282 \\ -0.662\text{x}10^{-1} & 0.629\text{x}10^{-1} & 0.282 & 0.149 \end{bmatrix}$$

Optimal polarizations : (Fig.4.17)

a) COPOL nulls :

$\rho_1^{co}=-0.2212-j0.4898$, $\theta_1^{co}=139.467°$, $\phi_1^{co}=-31.889°$

$\rho_2^{co}=0.4149+j0.563$, $\theta_2^{co}=40.877°$, $\phi_2^{co}=58.378°$

b) XPOL nulls :

$\rho_1^{x}=-8.4809+j0.1551$, $\theta_1^{x}=89.756°$, $\phi_1^{x}=-166.555°$

$\rho_2^{x}=0.1179-j0.2156$, $\theta_2^{x}=90.244°$, $\phi_2^{x}=13.445°$

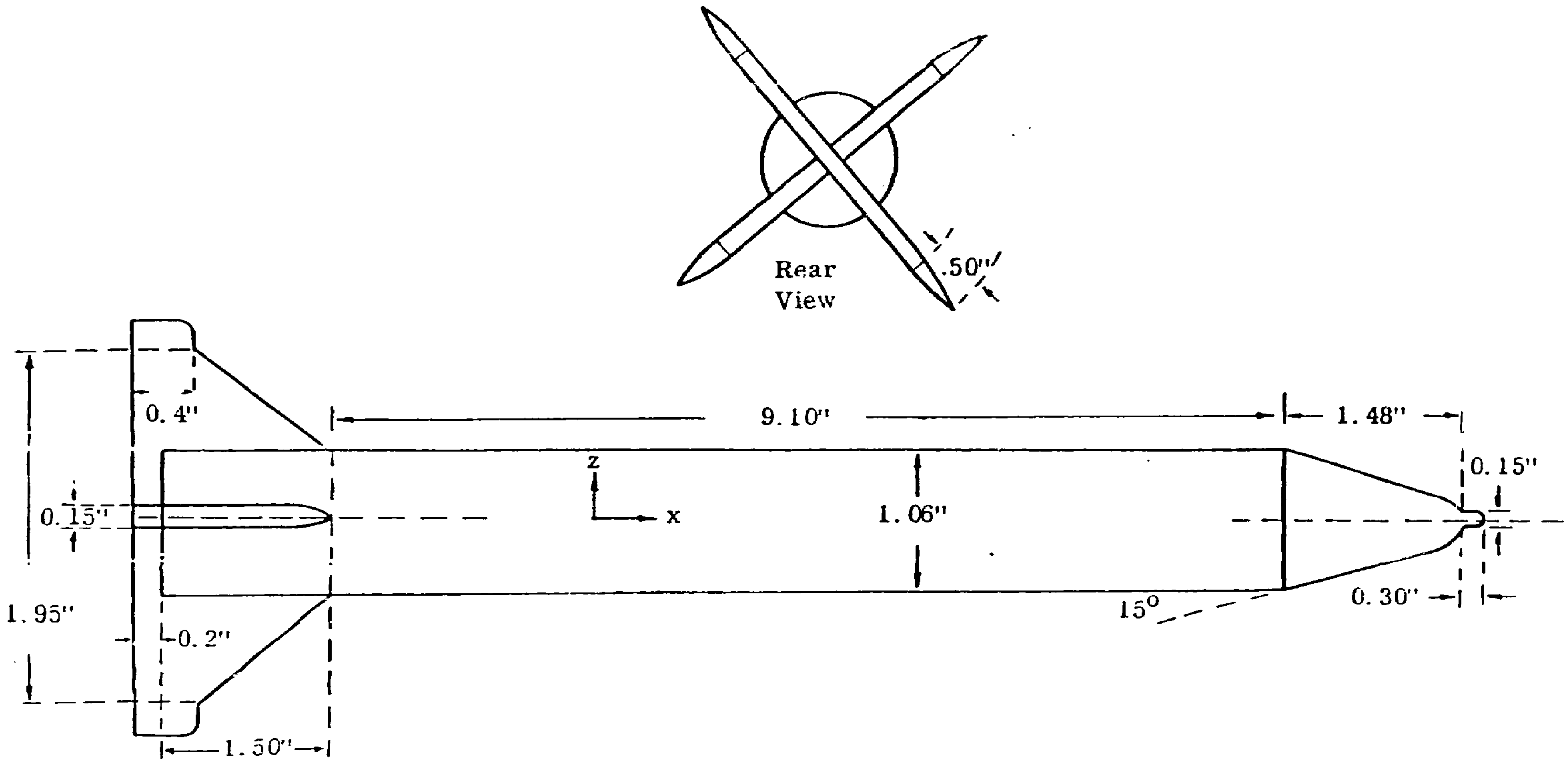


Fig. 4.16 : Sketch of Missile Model (Showing Fins , Crispin et al 1961).

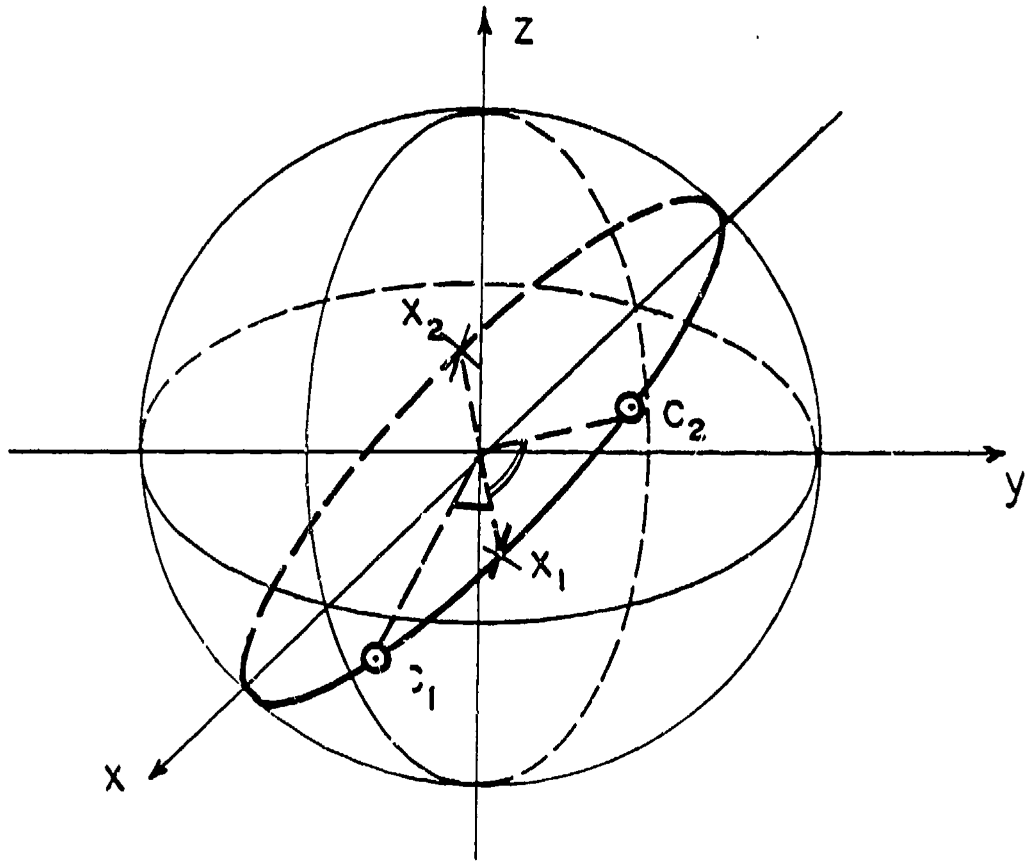


Fig. 4.17 : COPOL and XPOL nulls for a missile model with fins.

Example (8): Missile model without fins :

The geometry of this model is the same as the last one but without fins (Fig.4.16). The scattering matrix is calculated from the data given in Table 4.1 at aspect angle 180°. The calculated scattering matrix relative to a metallic sphere with diameter 1.98" is given by :

$$[S] = \begin{bmatrix} 0.569+j0.391 & 0.111\times10^{-1} \\ 0.111\times10^{-1} & 0.592+j0.415 \end{bmatrix}$$

and the Mueller matrices are :

$$[M] = \begin{bmatrix} 0.5 & -0.23\times10^{-1} & 0.129\times10^{-1} & -0.261\times10^{-3} \\ -0.23\times10^{-1} & 0.5 & -0.257\times10^{-3} & 0.892\times10^{-2} \\ 0.129\times10^{-1} & -0.257\times10^{-3} & 0.499 & -0.436\times10^{-2} \\ 0.261\times10^{-3} & -0.892\times10^{-2} & 0.436\times10^{-2} & 0.499 \end{bmatrix}$$

$$[Mm] = \begin{bmatrix} 0.477 & 0.123\times10^{-3} & 0.63\times10^{-2} & 0.433\times10^{-2} \\ 0.123\times10^{-3} & 0.523 & 0.656\times10^{-2} & -0.459\times10^{-2} \\ 0.126\times10^{-1} & 0.131\times10^{-1} & 0.499 & -0.436\times10^{-2} \\ -0.866\times10^{-2} & 0.918\times10^{-2} & 0.436\times10^{-2} & 0.499 \end{bmatrix}$$

Optimal polarizations : (Fig.4.18)

a) COPOL nulls :

$\rho_1^{co}=-0.1669\times10^{-1}-j0.9684$, $\theta_1^{co}=177.921°$, $\phi_1^{co}=-28.335°$

$\rho_2^{co}=-0.839\times10^{-1}+j0.986$, $\theta_2^{co}=0.934°$, $\phi_2^{co}=-31.12°$

b) XPOL nulls :

$\rho_1^{x}=3.837-j0.078$, $\theta_1^{x}=90.568°$, $\phi_1^{x}=150.796°$

$\rho_2^{x}=-0.2605+j0.5298$, $\theta_2^{x}=89.431°$, $\phi_2^{x}=-29.204°$

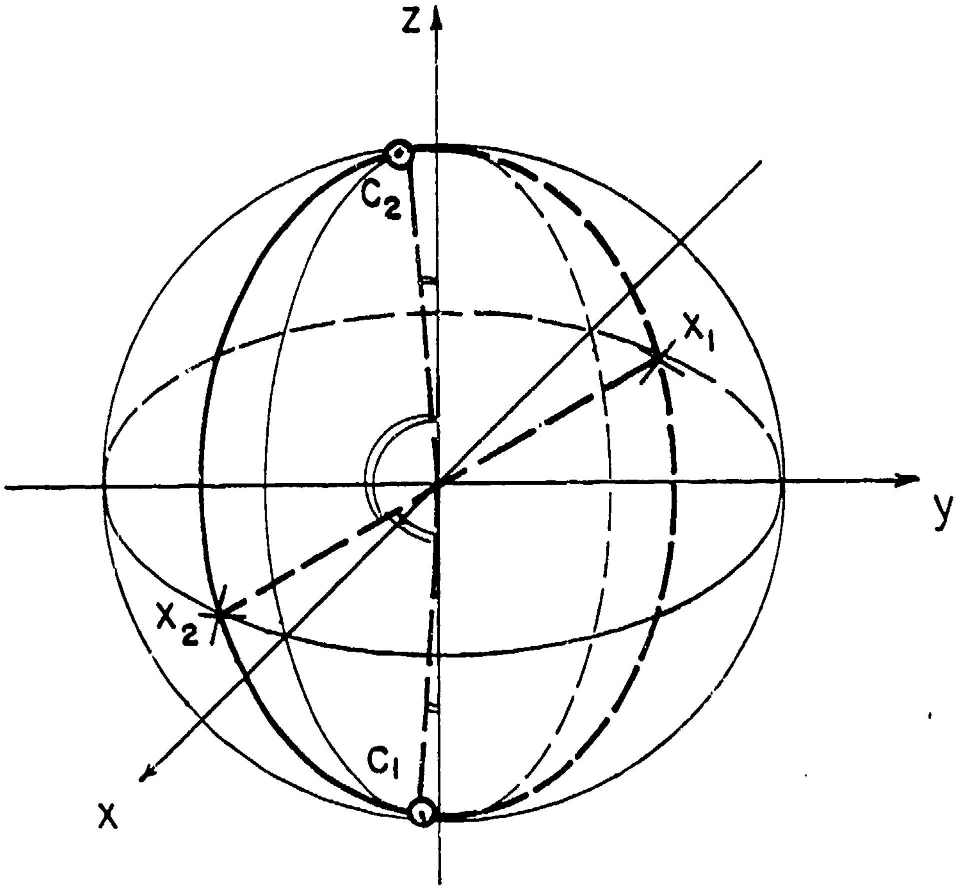


Fig. 4.18 : COPOL and XPOL nulls for a missile model without fins.

4.4 Sea clutter :

Example (9): Sea clutter and noise : (Fig.4.19)

The data of this example is taken from (Daley 1978) in which he based his calculations for the sea clutter scattering matrix on (Valonzuela 1968). The parameters in this case are given by :

a) dielectric constant of the sea water	=70-j70
b) wind velocity	=10 m/sec
c) tilt angle (for slightly rough surface)	$=4°$
d) depression angle	$=10°$
e) aspect angle	$=120°$
f) propagation constant	$=68.30°$
g) noise level	=-89.1db

the scattering matrix is given by :

$$[S] = \begin{bmatrix} 0.347\times10^{-1}+j0.206\times10^{-1} & 0.686\times10^{-1}+j0.737\times10^{-3} \\ 0.686\times10^{-1}+j0.737\times10^{-3} & 0.951+j0.291 \end{bmatrix}$$

where according to Section 3.6.1

$$[M] = \begin{bmatrix} 0.5 & -0.494 & 0.679\times10^{-1} & -0.179\times10^{-1} \\ -0.494 & 0.491 & -0.631\times10^{-1} & 0.206\times10^{-1} \\ 0.679\times10^{-1} & -0.631\times10^{-1} & 0.436\times10^{-1} & 0.946\times10^{-2} \\ 0.179\times10^{-1} & -0.206\times10^{-1} & -0.946\times10^{-2} & 0.342\times10^{-1} \end{bmatrix}$$

and

$$[Mm] = \begin{bmatrix} 0.162\times10^{-2} & 0.471\times10^{-2} & 0.239\times10^{-2} & 0.138\times10^{-2} \\ 0.471\times10^{-2} & 0.989 & 0.655\times10^{-1} & -0.193\times10^{-1} \\ 0.479\times10^{-2} & 0.131 & 0.436\times10^{-1} & 0.946\times10^{-2} \\ -0.277\times10^{-2} & 0.385\times10^{-1} & -0.946\times10^{-2} & 0.342\times10^{-1} \end{bmatrix}$$

Optimal polarizations : (Fig.4.20)

a) COPOL nulls :

$\rho_1^{co}=-0.3436\times10^{-1}-j0.1713,\quad \theta_1^{co}=109.414°,\quad \phi_1^{co}=-4.055°$

$\rho_2^{co}=-0.9805\times10^{-1}-j0.2102,\quad \theta_2^{co}=66.486°,\quad \phi_2^{co}=-11.708°$

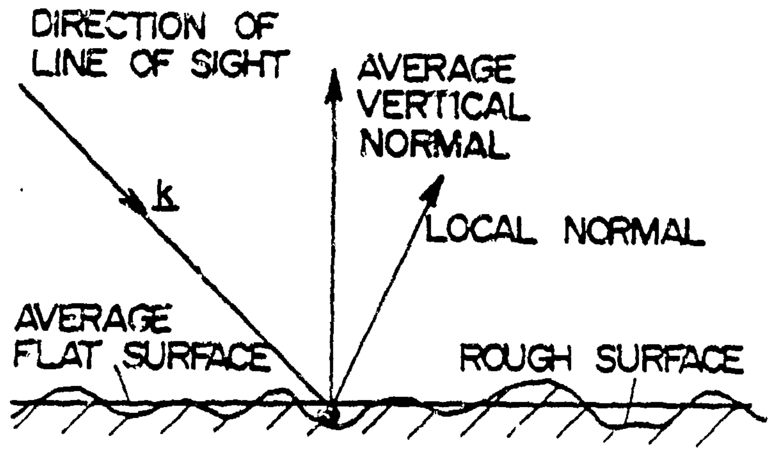


Fig. 4.19 : Geometry of a rough surface.

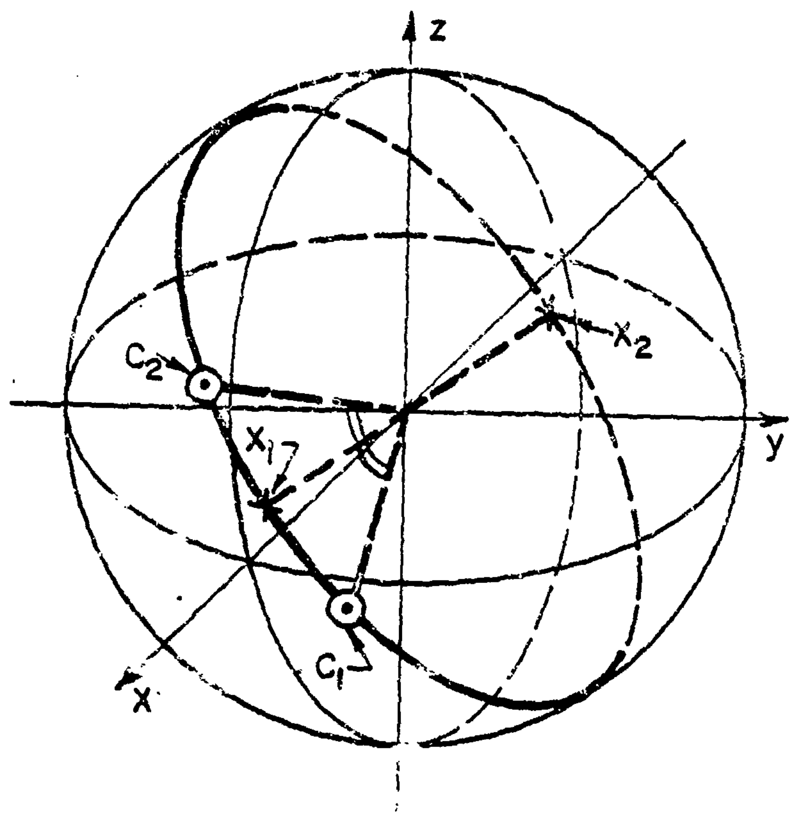


Fig. 4.20 : COPOL and XPOL nulls for sea clutter.

b) XPOL nulls :

$$\rho_1{}^x = 13.6727 - j3.6011, \qquad \theta_1{}^x = 92.054°, \qquad \phi_1{}^x = 172.172°$$

$$\rho_2{}^x = -0.6839\times10^{-1} + j0.1801\times10^{-1}, \qquad \theta_2{}^x = 87.946°, \qquad \phi_2{}^x = -7.828°$$

Example (10): Sea clutter, target and noise :

In this example, the conditions of the sea clutter and system noise are the same as in the example(9) but a simulated target is present and no target-clutter interaction is assumed. The scattering cross-sections of the simulated target are given by (Daley 1978) :

$$\sigma_{hh}(\text{target}) = \sigma_{hh}(\text{clutter})$$

$$\sigma_{vv}(\text{target}) = \sigma_{hh} - 3\text{db}$$

$$\sigma_{hv}(\text{target}) = \sigma_{hh} - 10\text{db}$$

and the phases are calculated randomly.

The total scattering matrix in this case for the same parameters of example (9) is given by :

$$[S] = \begin{bmatrix} 0.161+j0.69\text{x}10^{-1} & 0.257-j0.171\text{x}10^{-3} \\ 0.257-j0.171\text{x}10^{-3} & 0.817+j0.412 \end{bmatrix}$$

The Mueller matrix is :

$$[M] = \begin{bmatrix} 0.5 & -0.403 & 0.251 & -0.884\text{x}10^{-1} \\ -0.403 & 0.368 & -0.169 & 0.124 \\ 0.251 & -0.169 & 0.226 & -0.99\text{x}10^{-2} \\ -0.884\text{x}10^{-1} & -0.124 & 0.99\text{x}10^{-2} & 0.935\text{x}10^{-1} \end{bmatrix}$$

The modified Mueller matrix [Mm] is given by :

$$[Mm] = \begin{bmatrix} 0.306\text{x}10^{-1} & 0.662\text{x}10^{-1} & 0.413\text{x}10^{-1} & 0.178\text{x}10^{-1} \\ 0.662\text{x}10^{-1} & 0.837 & 0.210 & -0.106 \\ 0.826\text{x}10^{-1} & 0.420 & 0.226 & -0.990\text{x}10^{-2} \\ -0.355\text{x}10^{-1} & 0.212 & 0.990\text{x}10^{-2} & 0.935\text{x}10^{-1} \end{bmatrix}$$

Optimal polarizations (Fig. 4.21) :

a) COPOL nulls :

ρ_1^{co}=-0.1836-j0.2583, θ_1^{co}=118.001°, ϕ_1^{co}=-22.209°

ρ_2^{co}=-0.3182+j0.5121, θ_2^{co}=41.311°, ϕ_2^{co}=-45.001°

b) XPOL nulls :

ρ_1^{x}=3.1385-j1.1042, θ_1^{x}=100.543°, ϕ_1^{x}=148.062°

ρ_2^{x}=-0.2835+j0.9975x10^{-1}, θ_2^{x}=79.457°, ϕ_2^{x}=-31.938°

In the above two examples,there is a difference in two elements of the modified Mueller matrix, e.g. M_{31} and M_{41} as compared to Daley's results due to a printing mistake in Ishimaru's book (1978, vol.1, pp.35) and the correct elements should be according to Appendix (F) M_{31}=2Re{$S_{AA}S_{BA}^*$}, M_{41}=-2Im{$S_{AA}S_{BA}^*$}. In Daley's results, he used S_{BA} instead of S_{BA}^*. Also, it should be noted that some of the elements of [Mm] differ in sign with the ones mentioned in (Ishimaru 1978) and (Daley 1978) due to the difference in sign of g_{m3} in Eq.(3.9). Also, we noticed that, the XPOL nulls are antipodal and they should lie on a major circle with the COPOL nulls and bisect the arc between them. But in our model example the XPOL nulls are shifed with very small angles in both example which may be due to the fact that the scattering matrix does not completely satisfy the relative phase concept or due to some measurement errors which requires further analysis. For example, the scattering matrix in the first example has 0.62° absolute phase and for the second one 0.04°. We emphasize here the potential use of calculating the COPOL and XPOL nulls from measurement data immediately during measurement campaigns for the purpose of checking the accuracy of the measurements. This aspect is further discussed in Section 4.5; and will be treated in all detail in a forthcoming report.

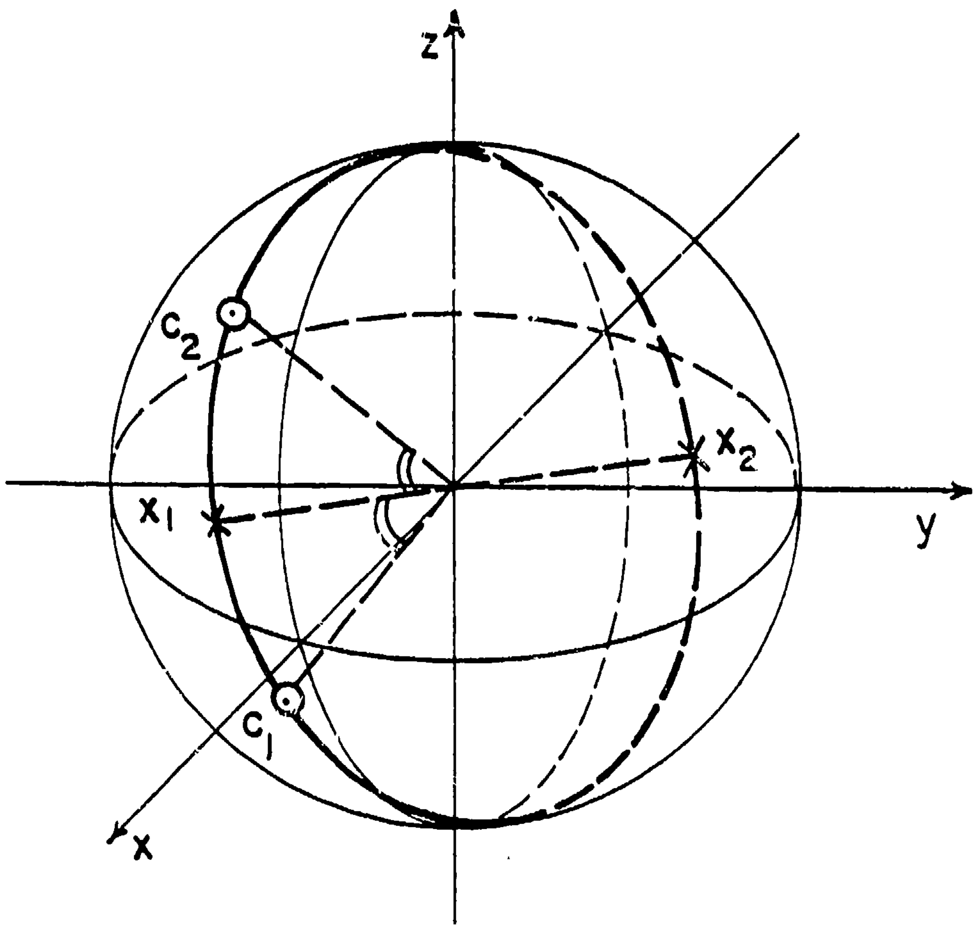


Fig. 4.21 : COPOL and XPOL nulls for a simulated target and sea clutter.

4.5 Discussions :

The numerical investigation of some target shapes and sea clutter has shown the feasibility of applying the theory of Chapter Three to calculate the two unknown matrices of the three matrices [S], [M] and [Mm] if one of them is known. We concentrated on the monostatic case only which is of specific interest here. It should be noted that, the scattering matrix with relative phase can be reconstructed from [M] or [Mm]. There are seven elements required for calculating [S] which are m_{11}, m_{12}, m_{13}, m_{14}, m_{22}, m_{23} and m_{24}.

For calculating the optimal polarization in the previous examples it is noticed that, as expected, the cross-polarization (XPOL) nulls are antipodal on the Poincare sphere and they bisect the angles between the co-polarization (COPOL) nulls. Also, it should be noted that [M] and [Mm] have only seven independent elements as shown in the calculations for the monostatic relative phase case as shown in Section 3.6.1.

More studies are needed for analyzing the same shapes of targets and others, and also for sea clutter with and without target to see the effect of changing the frequency or aspect angle on the COPOL and XPOL null locations and how they are moving on the Poincare sphere. For the mere reason to keep this report of still modest size, aspects of target optimal polarization characteristics will be treated in detail in a forthcoming report.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

We have thoroughly reviewed the literature on the basic theory of polarization utilization in radar target allocation, detection, imaging and identification. Based on this study, we decided to develop the theories and computer assisted numerical algorithms, step by step, rechecking every alternate representation found in the literature and thus correcting several misrepresented formulations. We will retain this approach throughout our studies and produce four major interim reports annually.

5.1 Progress Reported in this Report(January 15, 1981)

We rederived the basic formulations of polarization representation for the radar case. We established the relationship between the 2x2 radar scattering matrix[S], the 4x4 Stokes reflection matrix[M], and the 4x4 modified Mueller matrix[Mm], and vice versa for both the monostatic and the bistatic cases. We derived the associated expressions for calculating the CO-POLARIZATION (COPOL) as well as CROSS-POLARIZATION (XPOL) nulls and their presentations on the Poincare sphere. Inversely, given the radius p=span{[S]} of the Poincare sphere and the coordinates of either both COPOL nulls, or, of one COPOL and one XPOL null, we derived the expressions for the elements of [S], [M] and/or [Mm] re-expressed as functions of p and the coordinates of the two respectively given nulls.

We have developed computer program algorithms for calculating these scattering matrices, the associated optimal polarizations for the monostatic and bistatic cases for a variety of perfectly conducting target shapes. We have initiated close collaboration with several research laboratories in possession of excellent measurement facilities for obtaining reliable monostatic as well as bistatic scattering data, and some of the data made available have been used to calculate the respective optimal polarizations.

We have extended the mono-chromatic theory of the optimal polarization concept to the quasi-coherent and pan-chromatic cases, and we are completing a study on a novel vector scattering approach of extracting the useful target signal from clutter perturbed data which will be presented in detail in one of the forthcoming reports.

5.2 Conclusions

In analyzing the model data used for verification of the optimal polarization concept, it becomes very evident that we require to measure in the monostatic case the relative phases and magnitudes of the elements of the scattering matrix [S] to describe the complete electromagnetic properties of a target uniquely. For example (see Section 4.2), if the relative phase between the two co-polarized components, S_{AA} and S_{BB}, is not known, we find that for grossly different shapes the magnitudes can be identical resulting in ambiguity of identification (Section 4.2, examples 1, 2 and 5). Thus, the relative phases existing between all elements of the scattering matrix $[S]_{SMR}$ need to be measured for proper unique presentation of the target polarization characteristics which is being clearly verified by the examples presented in Section 4.2.

We also note that the inverse process of calculating the elements of [S], [M] and/or [Mm] from the coordinates of the respective optimal polarization nulls, provides deep inside into target characteristic properties and we should have a powerful tool for introducing the concept of target polarization synthesis. In target polarization synthesis, we would wish to design the shape and properties of a target such that it produces a given set of optimal polarization pairs.

5.3 Recommendations

Based upon the results of our model verification studies, we conclude that it is meritorious to further advance the optimal polarization concept first introduced by Professor Edward M. Kennaugh, subsequently extended by Dr. J. Huynen, and to utilize its great potential in radar target imaging and identification.

We also recommend that in microwave remote sensing using both passive (radiometry) and active (SAR, SLAR, SCATTEROMETRY) methods, measurements of the complete monostatic, relative phase scattering matrix[S] are made, i.e. both amplitudes and , the relative phases of all four (three) elements need to be executed to obtain unique information on the scattering properties of remotely sensed targets. The associated questions of applicable measurement techniques will be treated in a forthcoming report in great detail.

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Appendix (A)

Proof of Eqs.(3-8)

From Eqs.(3.3) and (3.7), we have :

$$\underline{h} = \begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} a_x e^{j\delta_x} \\ a_y e^{j\delta_y} \end{bmatrix} = a \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\tau \\ j\sin\tau \end{bmatrix} e^{j\alpha} \qquad \text{(A-1)}$$

Then, we can write h_x and h_y as :

$$h_x = a_x e^{j\delta_x} = a(\cos\phi\cos\tau - j\sin\phi\sin\tau)e^{j\alpha} \qquad \text{(A-2)}$$

and

$$h_y = a_y e^{j\delta_y} = a(\sin\phi\cos\tau + j\cos\phi\sin\tau)e^{j\alpha} \qquad \text{(A-3)}$$

From (A-2) and (A-3) :

$$|\underline{h}|^2 = |h_x|^2 + |h_y|^2$$

$$= a_x^2 + a_y^2$$

$$= a^2\{\cos^2\phi\cos^2\tau + \sin^2\phi\sin^2\tau + \sin^2\phi\cos^2\tau + \cos^2\phi\sin^2\tau\}$$

$$= a^2\{\cos^2\phi(\cos^2\tau + \sin^2\tau) + \sin^2\phi(\cos^2\tau + \sin^2\tau)\}$$

$$= a^2$$

Then

$$a_x^2 + a_y^2 = a^2 \qquad \text{(A-4)}$$

Also, from (A-2) and (A-3) :

$$h_x h_y^* = a_x a_y e^{j(\delta_x - \delta_y)} = a_x a_y e^{-j\delta} = a_x a_y(\cos\delta - j\sin\delta)$$

$$= a^2(\cos\phi\cos\tau - j\sin\phi\sin\tau)(\sin\phi\cos\tau - j\cos\phi\sin\tau)$$

Then

$$\mathrm{Re}\{h_x h_y^*\} = a_x a_y \cos\delta = a^2\{\cos\phi\sin\phi\cos^2\tau - \cos\phi\sin\phi\sin^2\tau\}$$

$$= \tfrac{1}{2}a^2\sin 2\phi(\cos^2\tau - \sin^2\tau)$$

$$= \tfrac{1}{2}a^2\sin 2\phi\cos 2\tau$$

Therefore, $2\mathrm{Re}\{h_x h_y^*\} = 2a_x a_y\cos\delta = a^2\sin2\phi\cos2\tau$ (A-5)

also,

$$-2\mathrm{Im}\{h_x h_y^*\} = 2a_x a_y\sin\delta$$

$$= 2a^2\{\cos^2\phi\sin\tau\cos\tau+\sin^2\phi\sin\tau\cos\tau\}$$

$$= 2a^2\sin\tau\cos\tau(\cos^2\phi+\sin^2\phi)$$

$$= a^2\sin2\tau$$

Therefore, $-2\mathrm{Im}\{h_x h_y^*\} = 2a_x a_y\sin\delta = a^2\sin2\tau$ (A-6)

From (A-6) we can write

$$\sin2\tau = \frac{2a_x a_y}{a^2}\sin\delta = \frac{2a_x a_y}{a_x^2+a_y^2}\sin\delta \qquad \text{(A-7)}$$

Also from (A-2) and (A-3), we can write :

$$|h_x|^2-|h_y|^2 = a_x^2-a_y^2 = a^2(\cos^2\phi\cos^2\tau+\sin^2\phi\sin^2\tau$$
$$-\sin^2\phi\cos^2\tau-\cos^2\phi\sin^2\tau\}$$

Then

$$a_x^2-a_y^2 = a^2(\cos^2\tau(\cos^2\phi-\sin^2\phi)-\sin^2\tau(\cos^2\phi-\sin^2\phi)\}$$

$$= a^2\cos2\phi(\cos^2\tau-\sin^2\tau)$$

$$= a^2\cos2\phi\cos2\tau$$

Therefore $a_x^2-a_y^2 = a^2\cos2\phi\cos2\tau$ (A-8)

Divide (A-5) onto (A-8), then

$$\tan2\phi = \frac{2a_x a_y}{a_x^2-a_y^2}\cos\delta \qquad \text{(A-9)}$$

In summary, from Eqs.(A-4), (A-7) and (A-9) we can rewrite them again :

$$a^2 = a_x^2+a_y^2$$

$$\tan 2\phi = \frac{2a_x a_y}{a_x^2-a_y^2}\cos\delta \tag{A-10}$$

$$\sin 2\tau = \frac{2a_x a_y}{a_x^2+a_y^2}\sin\delta$$

which are Eq.(3-8).

Appendix (B)

Stokes Parameters

From Eq.(3-9), the Stokes parameters are defined by :

$$
\begin{aligned}
g_0 &= |h_x|^2 + |h_y|^2 = I \\
g_1 &= |h_x|^2 - |h_y|^2 = Q \\
g_2 &= 2\mathrm{Re}(h_x h_y^*) \quad = U \\
g_3 &= -2\mathrm{Im}(h_x h_y^*) \quad = V
\end{aligned}
\tag{B-1}
$$

From Eqs.(A-4), (A-5), (A-6) and (A-8) in Appendix (A), we can rewrite the Stokes parameters in terms of $(a_x, a_y, \delta_x, \delta_y)$ or (a, ϕ, τ, α) as following :

$$
\begin{aligned}
g_0 &= |h_x|^2 + |h_y|^2 = a_x^2 + a_y^2 = a^2 && = I \\
g_1 &= |h_x|^2 - |h_y|^2 = a_x^2 - a_y^2 = a^2\cos 2\tau\cos 2\phi && = Q \\
g_2 &= 2\mathrm{Re}(h_x h_y^*) \quad = 2a_x a_y \cos\delta = a^2\cos 2\tau\sin 2\phi && = U \\
g_3 &= -2\mathrm{Im}(h_x h_y^*) \quad = 2a_x a_y \sin\delta = a^2\sin 2\tau && = V
\end{aligned}
\tag{B-2}
$$

From (B-2), we can write :

$$
\begin{aligned}
g_1^2 + g_2^2 + g_3^2 &= Q^2 + U^2 + V^2 \\
&= a^4(\cos^2 2\tau\cos^2 2\phi + \cos^2 2\tau\sin^2 2\phi + \sin^2 2\tau) \\
&= a^4(\cos^2 2\tau(\cos^2 2\phi + \sin^2 2\phi) + \sin^2 2\tau) \\
&= a^4(\cos^2 2\tau + \sin^2 2\tau) \\
&= a^4 = g_0^2 = I^2
\end{aligned}
$$

Then

$$
g_0^2 = g_1^2 + g_2^2 + g_3^2 = I^2 = Q^2 + U^2 + V^2 \tag{B-3}
$$

Appendix (C)

Representation of any polarization vector on Poincare's Sphere :

Given

$$\underline{h} = h_x \underline{h}_x + h_y \underline{h}_y \quad \text{(C-1)}$$

where h_x, h_y are in general complex numbers, and $\underline{h}_x$ and $\underline{h}_y$ are the unit vectors in x and y directions, respectively. If we introduce the auxiliary complex parameter u using h_x and h_y components of (C-1) then :

$$u = \frac{h_x - jh_y}{h_x + jh_y} = \frac{1-jP}{1+jP} \quad \text{(C-2)}$$

where

$$P = \frac{h_y}{h_x} = \text{polarization ratio} \quad \text{(C-3)}$$

But we have from Eq.(3-3) :

$$h_x = a_x e^{j\delta_x}, \quad h_y = a_y e^{j\delta_y} \quad \text{(C-4)}$$

where $a_x, a_y, \delta_x, \delta_y$ are real numbers.

Then :

$$P = \frac{h_y}{h_x} = \frac{a_y}{a_x} e^{j(\delta_y - \delta_x)} = \frac{a_y}{a_x} e^{j\delta} \quad \text{(C-5)}$$

where $\delta = \delta_y - \delta_x$ (C-6)

Substitute (C-5) into (C-2), then :

$$u = \frac{1-jP}{1+jP} = \frac{1-j(a_y/a_x)e^{j\delta}}{1+j(a_y/a_x)e^{j\delta}}$$

$$= \frac{1-j(a_y/a_x)\{\cos\delta+j\sin\delta\}}{1+j(a_y/a_x)\{\cos\delta+j\sin\delta\}}$$

Therefore

$$u = \frac{[1+(a_y/a_x)\sin\delta]-j(a_y/a_x)\cos\delta}{[1-(a_y/a_x)\sin\delta]+j(a_y/a_x)\cos\delta} \tag{C-7}$$

Introduce the real numbers X_1, X_2 and Y such that :

$$\begin{aligned} X_1 &= 1+(a_y/a_x)\sin\delta \\ X_2 &= 1-(a_y/a_x)\sin\delta \\ Y &= (a_y/a_x)\cos\delta \end{aligned} \tag{C-8}$$

Then :

$$u = \frac{X_1-jY}{X_2+jY} \tag{C-9}$$

From (C-9) :

$$|u|^2 = \frac{X_1{}^2+Y^2}{X_2{}^2+Y^2} = \frac{\{1+(a_y/a_x)\sin\delta\}^2+\{(a_y/a_x)\cos\delta\}^2}{\{1-(a_y/a_x)\sin\delta\}^2+\{(a_y/a_x)\cos\delta\}^2}$$

$$= \frac{1+2(a_y/a_x)\sin\delta+(a_y/a_x)^2}{1-2(a_y/a_x)\sin\delta+(a_y/a_x)^2}$$

Therefore :

$$\frac{|u|^2-1}{|u|^2+1} = \frac{\{1+2(a_y/a_x)\sin\delta+(a_y/a_x)^2\}-\{1-2(a_y/a_x)\sin\delta+(a_y/a_x)^2\}}{\{1+2(a_y/a_x)\sin\delta+(a_y/a_x)^2\}-\{1-2(a_y/a_x)\sin\delta+(a_y/a_x)^2\}}$$

$$= \frac{4(a_y/a_x)\sin\delta}{2[1+(a_y/a_x)^2]}$$

$$= \frac{2a_x a_y \sin\delta}{a_x^2+a_y^2}$$

$$= \frac{2a_x a_y \sin\delta}{a^2} \tag{C-10}$$

From Eqs.(B-2) Appendix B :

$$\begin{aligned} g_0 &= a^2 &&= a_x^2+a_y^2 \\ g_1 &= a^2\cos 2\tau\cos 2\phi &&= a_x^2 - a_y^2 \\ g_2 &= a^2\cos 2\tau\sin 2\phi &&= 2a_x a_y \cos\delta \\ g_3 &= a^2\sin 2\tau &&= 2a_x a_y \sin\delta \end{aligned} \tag{C-11}$$

Then :

$$\frac{g_3}{g_0} = \frac{a^2\sin 2\tau}{a^2} = \frac{2a_x a_y \sin\delta}{a^2}$$

$$\text{or} \quad \sin 2\tau = \frac{2a_x a_y \sin\delta}{a^2} \tag{C-12}$$

From (C-10) and (C-12), therefore

$$\sin 2\tau = \frac{|u|^2-1}{|u|^2+1} \tag{C-13}$$

If $\theta = \frac{1}{2}\pi - 2\tau$, therefore,

$$\cos\theta = \cos(\tfrac{1}{2}\pi-2\tau) = \sin 2\tau = \frac{|u|^2-1}{|u|^2+1}$$

Then, $\theta = \frac{1}{2}\pi - 2\tau = \arccos\{\frac{|u|^2-1}{|u|^2+1}\}$ (C-14)

From Eq.(C-9), we have :

$$u = \frac{X_1 - jY}{X_2 + jY} = \frac{X_1 - jY}{X_2 + jY}\,\frac{X_2 - jY}{X_2 - jY}$$

$$= \frac{(X_1X_2 - Y^2) - jY(X_1 + X_2)}{X_2{}^2 + Y^2}$$

$$\tan\{\text{phase}(u)\} = \frac{\text{Im}\{u\}}{\text{Re}\{u\}} = \frac{-Y(X_1 + X_2)}{X_1X_2 - Y^2} \quad \text{(C-15)}$$

Substitute from (C-8) into (C-15) :

$$\tan\{\text{phase}(u)\} = \frac{-2(a_y/a_x)\cos\delta}{[1+(a_y/a_x)\sin\delta][1-(a_y/a_x)\sin\delta]-[(a_y/a_x)\cos\delta]^2}$$

$$= \frac{-2(a_y/a_x)\cos\delta}{1-(a_y/a_x)^2\sin^2\delta-(a_y/a_x)^2\cos^2\delta}$$

$$= \frac{-2(a_y/a_x)\cos\delta}{1-(a_y/a_x)^2}$$

or

$$\tan\{\text{phase}(u)\} = \frac{-2a_xa_y\cos\delta}{a_x{}^2-a_y{}^2} \quad \text{(C-16)}$$

But from Eq.(C-11), we have :

$$\frac{g_2}{g_1} = \frac{2a_x a_y \cos\delta}{a_x^2 - a_y^2} = \frac{a^2 \cos 2\tau \sin 2\phi}{a^2 \cos 2\tau \cos 2\phi}$$

$$= \tan(2\phi) \qquad \text{(C-17)}$$

From Eqs.(C-16) and (C-17), therefore :

$$\tan(2\phi) = -\tan\{\text{phase}(u)\} \quad \text{or } 2\phi \approx -\text{phase}(u)$$

If we let $\phi' = 2\phi$, then

$$\phi' = 2\phi = -\arctan\{\frac{\text{Im}(u)}{\text{Re}(u)}\} = -\text{phase}(u) \qquad \text{(C-18)}$$

In summary :

If we have a polarization vector

$$h = h_x\hat{h}_x + h_y\hat{h}_y = \begin{bmatrix} a_x \\ a_y e^{j\delta} \end{bmatrix} e^{j\delta_x} ,$$

we can represent this vector on the Poincare sphere using the conventional spherical coordinate system (r, θ, ϕ') as follows :

If $u = \{(1-jP)/(1+jP)\}$ where P is the polarization ratio defined by : $P = h_y/h_x = (a_y/a_x)e^{j\delta}$, $\delta = \delta_y - \delta_x$,

therefore :

$$r = a^2 = a_x^2 + a_y^2$$

$$\theta = \tfrac{1}{2}\pi - 2\tau = \arccos\{\frac{|u|^2-1}{|u|^2+1}\} \qquad \text{(C-19)}$$

$$\phi' = 2\phi = -\text{phase}(u) = -\arctan\{\frac{\text{Im}(u)}{\text{Re}(u)}\}$$

Appendix D

Proof of : (I) Det{[S'(A',B')]} = Det{[S(A,B)]} = invariant

(II) Span{[S'(A',B')]} = Span{[S(A,B)]} = invariant

we have from (3.50) :

$$S'_{A'A'} = (1+\rho\rho^*)^{-1}[S_{AA}+\rho^2 S_{BB}+\rho(S_{AB}+S_{BA})] \tag{D-1}$$

$$S'_{A'B'} = (1+\rho\rho^*)^{-1}[-\rho^* S_{AA}+\rho S_{BB}+S_{AB}-\rho\rho^* S_{BA}] \tag{D-2}$$

$$S'_{B'A'} = (1+\rho\rho^*)^{-1}[-\rho^* S_{AA}+\rho S_{BB}+S_{BA}-\rho\rho^* S_{AB}) \tag{D-3}$$

$$S'_{B'B'} = (1+\rho\rho^*)^{-1}[\rho^{*2} S_{AA}+S_{BB}-\rho^*(S_{AB}+S_{BA})] \tag{D-4}$$

<u>Proof of (I) : Det{[S'(A',B')]} = Det{[S(A,B)]} = invariant :</u>

$$\mathrm{Det}\{[S'(A',B')]\} = S'_{A'A'}S'_{B'B'} - S'_{B'A'}S'_{A'B'}$$

$$= (1+\rho\rho^*)^{-2}\{[S_{AA}+\rho^2 S_{BB}+\rho(S_{AB}+S_{BA})][\rho^{*2}S_{AA}+S_{BB}-\rho^*(S_{AB}+S_{BA})]$$

$$-[-\rho^* S_{AA}+\rho S_{BB}+S_{AB}-\rho\rho^* S_{BA}][-\rho^* S_{AA}+\rho S_{BB}+S_{BA}-\rho\rho^* S_{AB}]\}$$

$$= (1+\rho\rho^*)^{-2}\{\rho^{*2}S_{AA}^2+\rho^2 S_{BB}^2-\rho\rho^*(S_{AB}+S_{BA})^2$$

$$+(1+\rho^{*2}\rho^2)S_{AA}S_{BB}+\rho^* S_{AA}(S_{AB}+S_{BA})(\rho\rho^*-1)+\rho S_{BB}(S_{AB}+S_{BA})(1-\rho^*\rho)$$

$$-\rho^{*2}S_{AA}^2+\rho\rho^* S_{AA}S_{BB}+\rho^* S_{AA}S_{BA}-\rho\rho^{*2}S_{AA}S_{AB}$$

$$+\rho\rho^* S_{AA}S_{BB}-\rho^2 S_{BB}^2-\rho S_{BA}S_{BB}+\rho^2\rho^* S_{AB}S_{BB}$$

$$+\rho^* S_{AA}S_{AB}-\rho S_{AB}S_{BB}-S_{AB}S_{BA}+\rho\rho^* S_{AB}^2$$

$$-\rho\rho^{*2}S_{AA}S_{BA}+\rho^2\rho^* S_{BA}S_{BB}-\rho^2\rho^{*2}S_{AB}S_{BA}+\rho\rho^* S_{BA}^2\}$$

$$= (1+\rho\rho^*)^{-2}\{S_{AA}S_{BB}(1+\rho^2\rho^{*2}+2\rho\rho^*)+S_{AB}S_{BA}(-2\rho\rho^*-1-\rho^2\rho^{*2})$$

$$+S_{AB}S_{AA}(-\rho^*+\rho\rho^{*2}-\rho\rho^{*2}+\rho^*)+S_{BA}S_{AA}(-\rho^*+\rho\rho^{*2}+\rho^*-\rho\rho^{*2})$$

$$+S_{AB}S_{BB}(-\rho^*\rho^2+\rho+\rho^2\rho^*-\rho)+S_{BA}S_{BB}(-\rho^*\rho^2+\rho-\rho+\rho^2\rho^*)\}$$

$$= (1+\rho\rho^*)^{-2}\{S_{AA}S_{BB}(1+\rho\rho^*)^2-S_{AB}S_{BA}(1+\rho\rho^*)^2\}$$

$$= \{S_{AA}S_{BB}-S_{AB}S_{BA}\}$$

$$= \det\{[S(A,B)]\}$$

Therefore, $\det\{[S'(A',B')]\} = \det\{[S(A,B)]\} =$ invariant (I)

<u>Proof of II : $\mathrm{Span}\{[S'(A',B')]\} = \mathrm{Span}\{[S(A,B)]\} =$ invariant</u>

From the definition :

$$\mathrm{Span}\ \{[S'(A',B')]\} = |S'_{A'A'}|^2+|S'_{A'B'}|^2+|S'_{B'A'}|^2+|S'_{B'B'}|^2$$

$$= S'_{A'A'}S'^*_{A'A'}+S'_{A'B'}S'^*_{A'B'}+S'_{B'A'}S'^*_{B'A'}+S'_{B'B'}S'^*_{B'B'} \quad \text{(D-}$$

Use Eqs.(D-1) to (D-4) and their conjugates into Eq.(D-5), therefore :

$$\mathrm{Span}\{[S'(A',B')]\} = \{[S_{AA}+\rho^2S_{BB}+\rho(S_{AB}+S_{BA})][S_{AA}^*+\rho^{*2}S_{BB}^*+\rho^*(S_{AB}^*+S_{BA}^*)]$$

$$+(-\rho^*S_{AA}+\rho S_{BB}+S_{AB}-\rho\rho^*S_{BA})(-\rho S_{AA}^*+\rho^*S_{BB}^*+S_{AB}^*-\rho\rho^*S_{BA}^*)$$

$$+(-\rho^*S_{AA}+\rho S_{BB}+S_{BA}-\rho\rho^*S_{AB})(-\rho S_{AA}^*+\rho^*S_{BB}^*+S_{BA}^*-\rho\rho^*S_{AB}^*)$$

$$+[\rho^{*2}S_{AA}+S_{BB}-\rho^*(S_{AB}+S_{BA})][\rho^2S_{AA}^*+S_{BB}^*-\rho(S_{AB}^*+S_{BA}^*)]\}\{(1+\rho\rho^*)$$

Therefore,

$$(1+\rho\rho^*)^2\mathrm{span}\{[S'(A',B')]\}$$

$$= S_{AA}S_{AA}^*[1+\rho\rho^*+\rho\rho^*+\rho^2\rho^{*2}]+S_{BB}S_{BB}^*[\rho^2\rho^{*2}+\rho\rho^*+\rho\rho^*+1]$$

$$+S_{AB}S_{AB}^*[\rho\rho^*+1+\rho^2\rho^{*2}+\rho\rho^*]+S_{BA}S_{BA}^*[\rho\rho^*+\rho^2\rho^{*2}+1+\rho\rho^*]$$

$$+S_{AA}S_{AB}^*[\rho^*-\rho^*+\rho\rho^{*2}-\rho\rho^{*2}]+S_{BB}S_{AB}^*[\rho^2\rho^*+\rho-\rho^2\rho^*-\rho]$$

$$+S_{AA}^{*}S_{AB}[\rho-\rho+\rho^{2}\rho^{*}-\rho^{*}\rho^{2}]+S_{BB}^{*}S_{AB}[\rho\rho^{*2}+\rho^{*}-\rho\rho^{*2}-\rho^{*}]$$

$$+S_{AA}S_{BA}^{*}[\rho^{*}+\rho\rho^{*2}-\rho^{*}-\rho\rho^{*2}]+S_{BB}S_{BA}^{*}[\rho^{2}\rho^{*}-\rho^{2}\rho^{*}+\rho-\rho]$$

$$+S_{AA}^{*}S_{BA}[\rho+\rho^{2}\rho^{*}-\rho-\rho^{*}\rho^{2}]+S_{BB}^{*}S_{BA}[\rho\rho^{*2}-\rho^{*2}\rho+\rho^{*}-\rho^{*}]$$

$$+S_{AA}S_{BB}^{*}[\rho^{*2}-\rho^{*2}-\rho^{*2}+\rho^{*2}]+S_{AB}S_{BA}^{*}[\rho\rho^{*}-\rho\rho^{*}-\rho\rho^{*}+\rho\rho^{*}]$$

$$+S_{AA}^{*}S_{BB}[\rho^{2}-\rho^{2}-\rho^{2}+\rho^{2}]+S_{AB}^{*}S_{BA}[\rho\rho^{*}-\rho\rho^{*}-\rho\rho^{*}+\rho\rho^{*}]$$

$$= (1+\rho\rho^{*})^{2}\{|S_{AA}|^{2}+|S_{BB}|^{2}+|S_{AB}|^{2}+|S_{BA}|^{2}\}$$

$$= (1+\rho\rho^{*})\mathrm{span}\{[S(A,B)]\}$$

Therefore

$$\mathrm{Span}\{[S'(A',B')]\} = \mathrm{span}\{[S(A,B)]\} = \mathrm{invariant} \qquad \text{(II)}$$

Appendix E

Derivation of the Mueller Matrix [M] from the Scattering Matrix [S]

Given

$$\underline{h} = h_A \hat{h}_A + h_B \hat{h}_B \tag{E-1}$$

and $\underline{g} = (g_0, g_1, g_2, g_3)$ so that :

$$g_0 = I = |h_A|^2 + |h_B|^2$$

$$g_1 = Q = |h_A|^2 - |h_B|^2$$

$$g_2 = U = 2\mathrm{Re}\{h_A h_B^*\} \tag{E-2}$$

$$g_3 = V = -2\mathrm{Im}\{h_A h_B^*\}$$

satisfying

$$g_0^2 = g_1^2 + g_2^2 + g_3^2 = I^2 = Q^2 + U^2 + V^2 \tag{E-3}$$

the scattered and the incident Stokes vectors $\underline{g}^s$ and $\underline{g}^i$ are related to each other by the Mueller Matrix [M] :

$$\underline{g}^s(A,B) = [M]\underline{g}^i(A,B) \tag{E-4}$$

We have also :

$$\underline{h}^s = [S]\,\underline{h}^i\,, \quad \text{or}$$

$$\begin{bmatrix} h_A^s \\ h_B^s \end{bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \begin{bmatrix} h_A^i \\ h_B^i \end{bmatrix} \tag{E-5}$$

Therefore :

$$h_A^s = S_{AA} h_A^i + S_{AB} h_B^i$$

$$h_B^s = S_{BA} h_A^i + S_{BB} h_B^i \tag{E-6}$$

1. Calculation of g_0^s, g_1^s :

$$|h_A^s|^2 = h_A^s h_A^{s*}$$

$$= (S_{AA}h_A^i+S_{AB}h_B^i)(S_{AA}^*h_A^{i*}+S_{AB}^*h_B^{i*})$$

$$= S_{AA}S_{AA}^*|h_A^i|^2+S_{AB}S_{AB}^*|h_B^i|^2+S_{AA}S_{AB}^*h_A^ih_B^{i*}+S_{AB}S_{AA}^*h_A^{i*}h_B^i$$

$$= |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+S_{AA}S_{AB}^*h_A^ih_B^{i*}$$

$$+(S_{AA}S_{AB}^*h_A^ih_B^{i*})^*$$

Therefore,

$$|h_A^s|^2 = |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+2\mathrm{Re}\{S_{AA}S_{AB}^*h_A^ih_B^{i*}\} \qquad \text{(E-7)}$$

But we have :

$$S_{AA}S_{AB}^*h_A^ih_B^{i*} = [\mathrm{Re}\{S_{AA}S_{AB}^*\}+j\mathrm{Im}\{S_{AA}S_{AB}^*\}][\mathrm{Re}\{h_A^ih_B^{i*}\}+j\mathrm{Im}\{h_A^ih_B^{i*}\}]$$

$$= \mathrm{Re}\{S_{AA}S_{AB}^*\}\mathrm{Re}\{h_A^ih_B^{i*}\}-\mathrm{Im}\{S_{AA}S_{AB}^*\}\mathrm{Im}\{h_A^ih_B^{i*}\}$$

$$+j[\mathrm{Re}\{S_{AA}S_{AB}^*\}\mathrm{Im}\{h_A^ih_B^{i*}\}+\mathrm{Re}\{h_A^ih_B^{i*}\}\mathrm{Im}\{S_{AA}S_{AB}^*\}]$$

Therefore,

$$2\mathrm{Re}\{S_{AA}S_{AB}^*h_A^ih_B^{i*}\} = 2\mathrm{Re}\{S_{AA}S_{AB}^*\}\mathrm{Re}\{h_A^ih_B^{i*}\}-2\mathrm{Im}\{S_{AA}S_{AB}^*\}\mathrm{Im}\{h_A^ih_B^{i*}\} \qquad \text{(E-8)}$$

Substitute from (E-8) into (E-7), therefore :

$$|h_A^s|^2 = |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+2\mathrm{Re}\{S_{AA}S_{AB}^*\}\mathrm{Re}\{h_A^ih_B^{i*}\}$$

$$-2\mathrm{Im}\{S_{AA}S_{AB}^*\}\mathrm{Im}\{h_A^ih_B^{i*}\} \qquad \text{(E-9)}$$

$$|h_B^s|^2 = h_B^s h_B^{s*}$$

$$= (S_{BA}h_A^i+S_{BB}h_B^i)(S_{BA}^*h_A^{i*}+S_{BB}^*h_B^{i*})$$

$$= |S_{BA}|^2|h_A^{i}|^2+|S_{BB}|^2|h_B^{i}|^2+S_{BA}S_{BB}^{*}h_A^{i}h_B^{i*}+(S_{BA}S_{BB}^{*}h_A^{i}h_B^{i*})^{*}$$

$$= |S_{BA}|^2|h_A^{i}|^2+|S_{BB}|^2|h_B^{i}|^2+2\mathrm{Re}\{S_{BA}S_{BB}^{*}h_A^{i}h_B^{i*}\} \qquad \text{(E-10)}$$

Similiar to Equation (E-8), we can write :

$$2\mathrm{Re}\{S_{BA}S_{BB}^{*}h_A^{i}h_B^{i*}\} = 2\mathrm{Re}\{S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}-2\mathrm{Im}\{S_{BA}S_{BB}^{*}\}\mathrm{Im}\{h_A^{i}h_B^{i*}\}$$

(E-11)

Substitute from (E-11) into (E-10), then :

$$|h_B^{s}|^2 = |S_{BA}|^2|h_A^{i}|^2+|S_{BB}|^2|h_B^{i}|^2+2\mathrm{Re}\{S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}$$

$$-2\mathrm{Im}\{S_{BA}S_{BB}^{*}\}\mathrm{Im}\{(h_A^{i}h_B^{i*}\} \qquad \text{(E-12)}$$

From Eqs.(E-2), (E-9) and (E-12), we can write g_0^{s} and g_1^{s} as follows :

$$g_0^{s} = |h_A^{s}|^2 + |h_B^{s}|^2$$

$$= (|S_{AA}|^2+|S_{BA}|^2)|h_A^{i}|^2+(|S_{AB}|^2+|S_{BB}|^2)|h_B^{i}|^2$$

$$+2\mathrm{Re}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}-2\mathrm{Im}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\}\mathrm{Im}\{h_A^{i}h_B^{i}$$

(E-13)

use the identity :

$$ax+by = \tfrac{1}{2}(a+b)(x+y) + \tfrac{1}{2}(a-b)(x-y) \qquad \text{(E-14)}$$

Therefore.

$$g_0^{s} = \tfrac{1}{2}(|S_{AA}|^2+|S_{BA}|^2+|S_{AB}|^2+|S_{BB}|^2)(|h_A^{i}|^2+|h_B^{i}|^2)$$

$$+\tfrac{1}{2}(|S_{AA}|^2+|S_{BA}|^2-|S_{AB}|^2-|S_{BB}|^2)(|h_A^{i}|^2-|h_B^{i}|^2)$$

$$+2\mathrm{Re}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}$$

$$-2\mathrm{Im}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\}\mathrm{Im}\{h_A^{i}h_B^{i*}\} \qquad \text{(E-15)}$$

Using the definition (E-2) into (E-15) : Therefore,

$$g_0^{s} = m_{11}g_0^{i}+m_{12}g_1^{i}+m_{13}g_2^{i}+m_{14}g_3^{i} \tag{E-16}$$

where :

$$m_{11} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2+|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2-|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\} \tag{E-17}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^{*}+S_{BA}S_{BB}^{*}\}$$

Also from Eqs.(E-2,9,12) :

$$g_1^{s} = |h_A^{s}|^2 - |h_B^{s}|^2$$

$$= (|S_{AA}|^2-|S_{BA}|^2)|h_A^{i}|^2+(|S_{AB}|^2-|S_{BB}|^2)|h_B^{i}|^2$$

$$+2\mathrm{Re}\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}-2\mathrm{Im}\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}\mathrm{Im}\{h_A^{i}h_B^{i*}\}$$

$$\tag{E-18}$$

Using the identity (E-14) with (E-18), then :

$$g_1^{s} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2+|S_{AB}|^2-|S_{BB}|^2\}(|h_A^{i}|^2+|h_B^{i}|^2)$$

$$+\tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2-|S_{AB}|^2+|S_{BB}|^2\}(|h_A^{i}|^2-|h_B^{i}|^2)$$

$$+2\mathrm{Re}\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}\mathrm{Re}\{h_A^{i}h_B^{i*}\}$$

$$-2\mathrm{Im}\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}\mathrm{Im}\{h_A^{i}h_B^{i*}\}$$

$$= m_{21}g_0^{i}+m_{22}g_1^{i}+m_{23}g_2^{i}+m_{24}g_3^{i}$$

where :

$$m_{21} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2+|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2-|S_{AB}|^2+|S_{BB}|^2\} \tag{E-19}$$

$$m_{23} = Re\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}$$

$$m_{24} = Im\{S_{AA}S_{AB}^{*}-S_{BA}S_{BB}^{*}\}$$

2. Calculation of g_2^s and g_3^s :

$$h_A^s h_B^{s*} = (S_{AA}h_A^i+S_{AB}h_B^i)(S_{BA}^* h_A^{i*}+S_{BB}^* h_B^{i*})$$

$$= S_{AA}S_{BA}^*|h_A^i|^2+S_{AB}S_{BB}^*|h_B^i|^2+S_{AA}S_{BB}^* h_A^i h_B^{i*}+S_{AB}S_{BA}^* h_A^{i*}h_B^i$$

Therefore,

$$h_A^s h_B^{s*} = [Re\{S_{AA}S_{BA}^*\}+jIm\{S_{AA}S_{BA}^*\}]|h_A^i|^2$$

$$+[Re\{S_{AB}S_{BB}^*\}+jIm\{S_{AB}S_{BB}^*\}]|h_B^i|^2$$

$$+[Re\{S_{AA}S_{BB}^*\}+jIm\{S_{AA}S_{BB}^*\}][Re\{h_A^i h_B^{i*}\}+jIm\{h_A^i h_B^{i*}\}]$$

$$+[Re\{S_{AB}S_{BA}^*\}+jIm\{S_{AB}S_{BA}^*\}][Re\{h_A^i h_B^{i*}\}-jIm\{h_A^i h_B^{i*}\}] \quad \text{(E-20}$$

From (E-2) and (E-20) :

$$g_2^s = 2Re\{h_A^s h_B^{s*}\}$$

$$= 2[Re\{S_{AA}S_{BA}^*\}|h_A^i|^2+Re\{S_{AB}S_{BB}^*\}|h_B^i|^2$$

$$+Re\{h_A^i h_B^{i*}\}Re\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}-Im\{h_A^i h_B^{i*}\}Im\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}]$$

$$= Re\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}\{|h_A^i|^2+|h_B^i|^2\}$$

$$+Re\{S_{AA}S_{BA}^*-S_{AB}S_{BB}^*\}\{|h_A^i|^2-|h_B^i|^2\}$$

$$+2Re\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}Re\{h_A^i h_B^{i*}\}-2Im\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}Im\{h_A^i h_B^{i*}\}$$

$$= m_{31}g_0^i+m_{32}g_1^i+m_{33}g_2^i+m_{34}g_3^i$$

where

$$m_{31} = Re\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{32} = \mathrm{Re}\{S_{AA}S_{BA}{}^{*} - S_{AB}S_{BB}{}^{*}\} \qquad \text{(E-21)}$$

$$m_{33} = \mathrm{Re}\{S_{AA}S_{BB}{}^{*} + S_{AB}S_{BA}{}^{*}\}$$

$$m_{34} = \mathrm{Im}\{S_{AA}S_{BB}{}^{*} - S_{AB}S_{BA}{}^{*}\}$$

Also from (E-2) and (E-20) :

$$g_3{}^{s} = -2\mathrm{Im}[h_A{}^{s}h_B{}^{s*}]$$

$$= -2[\mathrm{Im}\{S_{AA}S_{BA}{}^{*}\}|h_A{}^{i}|^2 + \mathrm{Im}\{S_{AB}S_{BB}{}^{*}\}|h_B{}^{i}|^2$$

$$+\mathrm{Re}(h_A{}^{i}h_B{}^{i*})\mathrm{Im}\{S_{AA}S_{BB}{}^{*} + S_{AB}S_{BA}{}^{*}\} + \mathrm{Im}(h_A{}^{i}h_B{}^{i*})\mathrm{Re}\{S_{AA}S_{BB}{}^{*} - S_{AB}S_{BA}{}^{*}\}]$$

$$= -\mathrm{Im}\{S_{AA}S_{BA}{}^{*} + S_{AB}S_{BB}{}^{*}\}(|h_A{}^{i}|^2 + |h_B{}^{i}|^2)$$

$$-\mathrm{Im}\{S_{AA}S_{BA}{}^{*} - S_{AB}S_{BB}{}^{*}\}(|h_A{}^{i}|^2 - |h_B{}^{i}|^2)$$

$$-2\mathrm{Im}\{S_{AA}S_{BB}{}^{*} + S_{AB}S_{BA}{}^{*}\}\mathrm{Re}(h_A{}^{i}h_B{}^{i*}) - 2\mathrm{Re}\{S_{AA}S_{BB}{}^{*} - S_{AB}S_{BA}{}^{*}\}\mathrm{Im}(h_A{}^{i}h_B{}^{i*})$$

$$= m_{41}g_0{}^{i} + m_{42}g_1{}^{i} + m_{43}g_2{}^{i} + m_{44}g_3{}^{i}$$

where :

$$m_{41} = -\mathrm{Im}\{S_{AA}S_{BA}{}^{*} + S_{AB}S_{BB}{}^{*}\}$$

$$m_{42} = -\mathrm{Im}\{S_{AA}S_{BA}{}^{*} - S_{AB}S_{BB}{}^{*}\}$$

$$m_{43} = -\mathrm{Im}\{S_{AA}S_{BB}{}^{*} + S_{AB}S_{BA}{}^{*}\} \qquad \text{(E-22)}$$

$$m_{44} = \mathrm{Re}\{S_{AA}S_{BB}{}^{*} - S_{AB}S_{BA}{}^{*}\}$$

In summary, then the elements of the Mueller matrix m_{ij} can be written in terms of the elements of the scattering matrix as follows :

<u>(a) Bistatic case</u> :

$$m_{11} = \tfrac{1}{2}(|S_{AA}|^2 + |S_{BA}|^2 + |S_{AB}|^2 + |S_{BB}|^2)$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2-|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^*+S_{BA}S_{BB}^*\}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^*+S_{BA}S_{BB}^*\}$$

$$m_{21} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2+|S_{AB}|^2-|S_{BB}|^2\}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2-|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{23} = \mathrm{Re}\{S_{AA}S_{AB}^*-S_{BA}S_{BB}^*\}$$

$$m_{24} = \mathrm{Im}\{S_{AA}S_{AB}^*-S_{BA}S_{BB}^*\}$$

$$m_{31} = \mathrm{Re}\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{32} = \mathrm{Re}\{S_{AA}S_{BA}^*-S_{AB}S_{BB}^*\}$$

$$m_{33} = \mathrm{Re}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}$$

$$m_{34} = \mathrm{Im}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}$$

$$m_{41} = -\mathrm{Im}\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{42} = -\mathrm{Im}\{S_{AA}S_{BA}^*-S_{AB}S_{BB}^*\}$$

$$m_{43} = -\mathrm{Im}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}$$

$$m_{44} = \mathrm{Re}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\} \qquad \text{(E-23)}$$

b) <u>Monostatic case ($S_{AB}=S_{BA}$)</u> :

$$m_{11} = \tfrac{1}{2}\{|S_{AA}|^2+2|S_{AB}|^2+|S_{BB}|^2\}$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BB}|^2\}$$

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^*+S_{AB}S_{BB}^*\}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^*+S_{AB}S_{BB}^*\}$$

$$m_{21} = \tfrac{1}{2}\{|S_{AA}|^2 - |S_{BB}|^2\} = m_{12}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2 + |S_{BB}|^2 - 2|S_{AB}|^2\}$$

$$m_{23} = \mathrm{Re}\{S_{AA}S_{AB}{}^* - S_{AB}S_{BB}{}^*\}$$

$$m_{24} = \mathrm{Im}\{S_{AA}S_{AB}{}^* - S_{AB}S_{BB}{}^*\}$$

$$m_{31} = \mathrm{Re}\{S_{AA}S_{AB}{}^* + S_{AB}S_{BB}{}^*\} = m_{13}$$

$$m_{32} = \mathrm{Re}\{S_{AA}S_{AB}{}^* - S_{AB}S_{BB}{}^*\} = m_{23}$$

$$m_{33} = \mathrm{Re}\{S_{AA}S_{BB}{}^*\} + |S_{AB}|^2$$

$$m_{34} = \mathrm{Im}\{S_{AA}S_{BB}{}^*\}$$

$$m_{41} = -\mathrm{Im}\{S_{AA}S_{AB}{}^* + S_{AB}S_{BB}{}^*\} = -m_{14}$$

$$m_{42} = -\mathrm{Im}\{S_{AA}S_{AB}{}^* - S_{AB}S_{BB}{}^*\} = -m_{24}$$

$$m_{43} = -\mathrm{Im}\ \{S_{AA}S_{BB}{}^*\} = -m_{34}$$

$$m_{44} = \mathrm{Re}\{S_{AA}S_{BB}{}^*\} - |S_{AB}|^2 \qquad (\Sigma\text{-}24)$$

we also do notice that

$$m_{44} = m_{33} + m_{22} - m_{11}$$

Appendix F

Derivation of the Modified Mueller Matrix [Mm] from the Scattering Matrix[S] :

Define the modified Stokes parameters for the polarization vector

$$\underline{h} = h_A \hat{h}_A + h_B \hat{h}_B \tag{F-1}$$

as :

$$\begin{aligned} g_{m0} &= |h_A|^2 &&= I_A \\ g_{m1} &= |h_B|^2 &&= I_B \\ g_{m2} &= 2\mathrm{Re}\{h_A h_B^*\} &&= U \\ g_{m3} &= -2\mathrm{Im}\{h_A h_B^*\} &&= V \end{aligned} \tag{F-2}$$

and

$$\underline{g_m}^s(A,B) = [Mm]\underline{g_m}^i(A,B) \tag{F-3}$$

We have :

$$\begin{bmatrix} h_A{}^s \\ h_B{}^s \end{bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \begin{bmatrix} h_A{}^i \\ h_B{}^i \end{bmatrix} \tag{F-4}$$

Therefore,

$$\begin{aligned} h_A{}^s &= S_{AA} h_A{}^i + S_{AB} h_B{}^i \\ h_B{}^s &= S_{BA} h_A{}^i + S_{BB} h_B{}^i \end{aligned} \tag{F-5}$$

Then

$$\begin{aligned} I_A{}^s &= |h_A{}^s|^2 = h_A{}^s h_A{}^{s*} \\ &= (S_{AA} h_A{}^i + S_{AB} h_B{}^i)(S_{AA}{}^* h_A{}^{i*} + S_{AB}{}^* h_B{}^{i*}) \\ &= |S_{AA}|^2 |h_A{}^i|^2 + |S_{AB}|^2 |h_B{}^i|^2 + S_{AB} S_{AA}{}^* h_A{}^{i*} h_B{}^i + S_{AA} S_{AB}{}^* h_A{}^i h_B{}^{i*} \end{aligned}$$

Therefore,

$$I_A^s = |h_A^s|^2$$

$$= |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+(S_{AA}S_{AB}^*h_A^ih_B^{i*})+(S_{AA}S_{AB}^*h_A^ih_B^{i*})^*$$

$$= |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+2Re\{S_{AA}S_{AB}^*h_A^ih_B^{i*}\} \qquad \text{(F-6)}$$

But we have :

$$S_{AA}S_{AB}^*h_A^ih_B^{i*} = [Re\{S_{AA}S_{AB}^*\}+jIm\{S_{AA}S_{AB}^*\}][Re\{h_A^ih_B^{i*}\}+jIm\{h_A^ih_B^{i*}\}]$$

$$= [Re\{S_{AA}S_{AB}^*\}Re\{h_A^ih_B^{i*}\}-Im\{S_{AA}S_{AB}^*\}Im\{h_A^ih_B^{i*}\}]$$

$$+j[Im\{S_{AA}S_{AB}^*\}Re\{h_A^ih_B^{i*}\}+Re\{S_{AA}S_{AB}^*\}Im\{h_A^ih_B^{i*}\}]$$

Therefore,

$$2Re\{S_{AA}S_{AB}^*h_A^ih_B^{i*}\} = 2Re\{S_{AA}S_{AB}^*\}Re\{h_A^ih_B^{i*}\}-2Im\{S_{AA}S_{AB}^*\}Im\{h_A^ih_B^{i*}\} \qquad \text{(F-7)}$$

$$2Im\{S_{AA}S_{AB}^*h_A^ih_B^{i*}\} = 2Im\{S_{AA}S_{AB}^*\}Re\{h_A^ih_B^{i*}\}+2Re\{S_{AA}S_{AB}^*\}Im\{h_A^ih_B^{i*}\} \qquad \text{(F-8)}$$

Substitute from (F-7) into (F-6), therefore :

$$I_A^s = |h_A^s|^2$$

$$= |S_{AA}|^2|h_A^i|^2+|S_{AB}|^2|h_B^i|^2+2Re\{S_{AA}S_{AB}^*\}Re\{h_A^ih_B^{i*}\}$$

$$-2Im\{S_{AA}S_{AB}^*\}Im\{h_A^ih_B^{i*}\}$$

Therefore,

$$I_A^s = |S_{AA}|^2I_A^i+|S_{AB}|^2I_B^i+Re\{S_{AA}S_{AB}^*\}U^i+Im\{S_{AA}S_{AB}^*\}V^i \qquad \text{(I)}$$

also we have :

$$I_B^s = |h_B^s|^2 = h_B^sh_B^{s*}$$

$$= (S_{BA}h_A^i+S_{BB}h_B^i)(S_{BA}^*h_A^{i*}+S_{BB}^*h_B^{i*})$$

$$= |S_{BA}|^2|h_A{}^i|^2+|S_{BB}|^2|h_B{}^i|^2+S_{BA}S_{BB}{}^*h_A{}^ih_B{}^{i*}+(S_{BA}S_{BB}{}^*h_A{}^ih_B{}^{i*})^*$$

$$= |S_{BA}|^2|h_A{}^i|^2+|S_{BB}|^2|h_B{}^i|^2+2Re\{S_{BA}S_{BB}{}^*\}Re\{h_A{}^ih_B{}^{i*}\}$$

$$-2Im\{S_{BA}S_{BB}{}^*\}Im\{h_A{}^ih_B{}^{i*}\}$$

Then

$$I_B{}^s = |S_{BA}|^2I_A{}^i+|S_{BB}|^2I_B{}^i+Re\{S_{BA}S_{BB}{}^*\}U^i+Im\{S_{BA}S_{BB}{}^*\}V^i \qquad \text{(II)}$$

also we have :

$$h_A{}^sh_B{}^{s*} = (S_{AA}h_A{}^i+S_{AB}h_B{}^i)(S_{BA}{}^*h_A{}^{i*}+S_{BB}{}^*h_B{}^{i*})$$

$$= S_{AA}S_{BA}{}^*|h_A{}^i|^2+S_{AB}S_{BB}{}^*|h_B{}^i|^2+S_{AA}S_{BB}{}^*h_A{}^ih_B{}^{i*}+S_{AB}S_{BA}{}^*(h_A{}^ih_B{}^{i*})$$

$$= Re\{S_{AA}S_{BA}{}^*\}|h_A{}^i|^2+Re\{S_{AB}S_{BB}{}^*\}|h_B{}^i|^2$$

$$+jIm\{S_{AA}S_{BA}{}^*\}|h_A{}^i|^2+jIm\{S_{AB}S_{BB}{}^*\}|h_B{}^i|^2$$

$$+[Re\{S_{AA}S_{BB}{}^*\}+jIm\{S_{AA}S_{BB}{}^*\}][Re\{h_A{}^ih_B{}^{i*}\}+jIm\{h_A{}^ih_B{}^{i*}\}]$$

$$+[Re\{S_{AB}S_{BA}{}^*\}+jIm\{S_{AB}S_{BA}{}^*\}][Re\{h_A{}^ih_B{}^{i*}\}-jIm\{h_A{}^ih_B{}^{i*}\}]$$

Then

$$2Re\{h_A{}^sh_B{}^{s*}\} = 2Re\{S_{AA}S_{BA}{}^*\}|h_A|^2+2Re\{S_{AB}S_{BB}{}^*\}|h_B|^2$$

$$+2Re\{h_A{}^ih_B{}^{i*}\}Re\{S_{AA}S_{BB}{}^*+S_{AB}S_{BA}{}^*\}$$

$$-2Im\{h_A{}^ih_B{}^{i*}\}Im\{S_{AA}S_{BB}{}^*-S_{AB}S_{BA}{}^*\}$$

Therefore,

$$U^s = 2Re\{S_{AA}S_{BA}{}^*\}I_A{}^i+2Re\{S_{AB}S_{BB}{}^*\}I_B{}^i$$

$$+Re\{S_{AA}S_{BB}{}^*+S_{AB}S_{BA}{}^*\}U^i+Im\{S_{AA}S_{BB}{}^*-S_{AB}S_{BA}{}^*\}V^i \qquad \text{(III)}$$

also,

$$-2\mathrm{Im}\{h_A{}^s h_B{}^{s*}\} = -2\mathrm{Im}\{S_{AA}S_{BA}{}^*\}|h_A{}^i|^2 - 2\mathrm{Im}\{S_{AB}S_{BB}{}^*\}|h_B{}^i|^2$$

$$-2\mathrm{Im}\{S_{AA}S_{BB}{}^* + S_{AB}S_{BA}{}^*\}\mathrm{Re}\{h_A{}^i h_B{}^{i*}\}$$

$$-2\mathrm{Re}\{S_{AA}S_{BB}{}^* - S_{AB}S_{BA}{}^*\}\mathrm{Im}\{h_A{}^i h_B{}^{i*}\}$$

Therefore,

$$V^s = -2\mathrm{Im}\{S_{AA}S_{BA}{}^*\}I_A{}^i - 2\mathrm{Im}\{S_{AB}S_{BB}{}^*\}I_B{}^i$$

$$-\mathrm{Im}\{S_{AA}S_{BB}{}^* + S_{AB}S_{BA}{}^*\}U^i + \mathrm{Re}\{S_{AA}S_{BB}{}^* - S_{AB}S_{BA}{}^*\}V^i \qquad \text{(IV)}$$

In summary, from (I) to (IV), we can write [Mm] as :

$$\left[M_m\right] = \begin{bmatrix} |S_{AA}|^2 & |S_{AB}|^2 & \mathrm{Re}\{S_{AA}S_{AB}{}^*\} & \mathrm{Im}\{S_{AA}S_{AB}{}^*\} \\ |S_{BA}|^2 & |S_{BB}|^2 & \mathrm{Re}\{S_{BA}S_{BB}{}^*\} & \mathrm{Im}\{S_{BA}S_{BB}{}^*\} \\ 2\mathrm{Re}\{S_{AA}S_{BA}{}^*\} & 2\mathrm{Re}\{S_{AB}S_{BB}{}^*\} & \mathrm{Re}\{S_{AA}S_{BB}{}^* + S_{AB}S_{BA}{}^*\} & \mathrm{Im}\{S_{AA}S_{BB}{}^* - S_{AB}S_{BA}{}^*\} \\ -2\mathrm{Im}\{S_{AA}S_{BA}{}^*\} & -2\mathrm{Im}\{S_{AB}S_{BB}{}^*\} & -\mathrm{Im}\{S_{AA}S_{BB}{}^* + S_{AB}S_{BA}{}^*\} & \mathrm{Re}\{S_{AA}S_{BB}{}^* - S_{AB}S_{BA}{}^*\} \end{bmatrix} \qquad \text{(V)}$$

Appendix G

Derivation of the Scattering Matrix [S] from the Mueller Matrix[M] :

It is required to calculate the scattering matrix [S] from the Mueller matrix[M] in the general bistatic and monostatic cases.

The scattering matrix [S] relates the scattering and the incident polarization vectors by the equation :

$$\underline{h}^{s}(A,B) = [S]\ \underline{h}^{i}(A,B) \tag{G-1}$$

where :

$$[S] = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} = \begin{bmatrix} |S_{AA}|e^{j\phi_{AA}} & |S_{AB}|e^{j\phi_{AB}} \\ |S_{BA}|e^{j\phi_{BA}} & |S_{BB}|e^{j\phi_{BB}} \end{bmatrix} \tag{G-2}$$

Also the Mueller matrix relates the scattered and the incident Stokes vectors by :

$$\underline{g}^{s}(A,B) = [M]\underline{g}^{i}(A,B) \tag{G-3}$$

where :

$$g = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |h_A|^2+|h_B|^2 \\ |h_A|^2-|h_B|^2 \\ 2\mathrm{Re}\{h_A h_B^*\} \\ -2\mathrm{Im}\{h_A h_B^*\} \end{bmatrix} \tag{G-4}$$

1. Calculation of the magnitudes $|S_{AA}|,|S_{AB}|,|S_{BA}|$ and $|S_{BB}|$:

From Appendix (E), we have :

$$m_{11} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2+|S_{AB}|^2+|S_{BB}|^2\} \tag{G-5}$$

$$m_{12} = \tfrac{1}{2}\{|S_{AA}|^2+|S_{BA}|^2-|S_{AB}|^2-|S_{BB}|^2\} \tag{G-6}$$

$$m_{21} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2+|S_{AB}|^2-|S_{BB}|^2\} \tag{G-7}$$

$$m_{22} = \tfrac{1}{2}\{|S_{AA}|^2-|S_{BA}|^2-|S_{AB}|^2+|S_{BB}|^2\} \tag{G-8}$$

we have :

$$m_{11} + m_{12} = |S_{AA}|^2 + |S_{BA}|^2 \tag{G-9}$$

$$m_{21} + m_{22} = |S_{AA}|^2 - |S_{BA}|^2 \tag{G-10}$$

then :

$$m_{11} + m_{12} + m_{21} + m_{22} = 2|S_{AA}|^2$$

and

$$m_{11} + m_{12} - m_{21} - m_{22} = 2|S_{BA}|^2 \text{ , therefore}$$

$$|S_{AA}| = \tfrac{1}{2}(m_{11}+m_{12}+m_{21}+m_{22}) \tag{G-11}$$

$$|S_{BA}| = \tfrac{1}{2}(m_{11}+m_{12}-m_{21}-m_{22}) \tag{G-12}$$

<u>Similiary</u> :

$$m_{11} - m_{12} = |S_{AB}|^2 + |S_{BB}|^2$$

$$m_{21} - m_{22} = |S_{AB}|^2 - |S_{BB}|^2$$

Therefore,

$$|S_{AB}| = \tfrac{1}{2}(m_{11}-m_{12}+m_{21}-m_{22}) \tag{G-13}$$

$$|S_{BB}| = \tfrac{1}{2}(m_{11}-m_{12}-m_{21}+m_{22}) \tag{G-14}$$

2. Calculations of the phases ϕ_{AA}, ϕ_{AB}, ϕ_{BA}, ϕ_{BB} :

Also, we have from Appandix (E) :

$$m_{13} = \mathrm{Re}\{S_{AA}S_{AB}^{*} + S_{BA}S_{BB}^{*}\}$$

$$m_{14} = \mathrm{Im}\{S_{AA}S_{AB}^{*} + S_{BA}S_{BB}^{*}\}$$

Then :

$$m_{13} + jm_{14} = S_{AA}S_{AB}^* + S_{BA}S_{BB}^* \qquad \text{(G-15)}$$

Also,

$$m_{23} = \mathrm{Re}\{S_{AA}S_{AB}^* - S_{BA}S_{BB}^*\}$$

$$m_{24} = \mathrm{Im}\{S_{AA}S_{AB}^* - S_{BA}S_{BB}^*\}$$

Therefore,

$$m_{23} + jm_{24} = S_{AA}S_{AB}^* - S_{BA}S_{BB}^* \qquad \text{(G-16)}$$

From (G-15) and (G-16) :

$$(m_{13}+jm_{14}) + (m_{23}+jm_{24}) = 2S_{AA}S_{AB}^*$$

or

$$(m_{13}+m_{23}) + j(m_{14}+m_{24}) = 2|S_{AA}||S_{AB}|e^{j(\phi_{AA}-\phi_{AB})}$$

Therefore,

$$\phi_{AA} = \phi_{AB}+\arctan\left(\frac{m_{14}+m_{24}}{m_{13}+m_{23}}\right) \qquad \text{(G-17)}$$

Also from (G-15) and (G-16) :

$$(m_{13}+jm_{14}) - (m_{23}+jm_{24}) = 2S_{BA}S_{BB}^*$$

$$(m_{13}-m_{23}) + j(m_{14}-m_{24}) = 2|S_{BA}||S_{BB}|e^{j(\phi_{BA}-\phi_{BB})}$$

$$\phi_{BA}-\phi_{BB} = \arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right) \qquad \text{(G-18)}$$

From Appendix E, we also have :

$$m_{31} = \mathrm{Re}\{S_{AA}S_{BA}^*+S_{AB}S_{BB}^*\}$$

$$m_{41} = -\mathrm{Im}\{S_{AA}S_{BA}^{*}+S_{AB}S_{BB}^{*}\}$$

$$m_{32} = \mathrm{Re}\{S_{AA}S_{BA}^{*}-S_{AB}S_{BB}^{*}\}$$

$$m_{42} = -\mathrm{Im}\{S_{AA}S_{BA}^{*}-S_{AB}S_{BB}^{*}\}$$

Therefore,

$$m_{31}-jm_{41} = S_{AA}S_{BA}^{*}+S_{AB}S_{BB}^{*} \tag{G-19}$$

and

$$m_{32}-jm_{42} = S_{AA}S_{BA}^{*}-S_{AB}S_{BB}^{*}\ , \text{ then} \tag{G-20}$$

$$(m_{31}-jm_{41})-(m_{32}-jm_{42}) = 2S_{AB}S_{BB}^{*}$$

$$(m_{31}-m_{32})+j(m_{42}-m_{41}) = 2|S_{AB}||S_{BB}|e^{j(\phi_{AB}-\phi_{BB})}\ , \text{ therefore}$$

$$\phi_{AB}-\phi_{BB} = \arctan\left(\frac{m_{42}-m_{41}}{m_{31}-m_{32}}\right)\ , \text{ or}$$

$$\phi_{BB} = \phi_{AB}+\arctan\left(\frac{m_{41}-m_{42}}{m_{31}-m_{32}}\right) \tag{G-21}$$

From (G-18), (G-21), therefore :

$$\phi_{BA} = \phi_{BB}+\arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right)$$

$$\phi_{BA} = \phi_{AB} + \arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right) + \arctan\left(\frac{m_{41}-m_{42}}{m_{31}-m_{32}}\right) \qquad \text{(G-22)}$$

In the monostatic case :

From Appendix (E), we have :

$$m_{21} = m_{12}$$

$$m_{31} = m_{13}$$

$$m_{32} = m_{23}$$

$$m_{41} = -m_{14}$$

$$m_{42} = -m_{24}$$

$$m43 = -m34$$

$$m44 = m33 + m_{22} - m_{11}$$

Therefore,

$$|S_{AA}| = \tfrac{1}{2}(m_{11}+2m_{12}+m_{22})$$

$$|S_{AB}| = \tfrac{1}{2}(m_{11}-m_{22}) = |S_{BA}|$$

$$|S_{BB}| = \tfrac{1}{2}(m_{11}-2m_{12}+m_{22})$$

$$\phi_{AA} = \phi_{AB} + \arctan\left(\frac{m_{14}+m_{24}}{m_{13}+m_{23}}\right)$$

ϕ_{AB} is arb.

$$\phi_{BA} = \phi_{AB} + \arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right) + \arctan\left(\frac{-m_{14}+m_{24}}{m_{13}-m_{23}}\right)$$

$$= \phi_{AB}$$

$$\phi_{BB} = \phi_{AB} - \arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right)$$

In summary :

1. Bistatic case :

$$|S_{AA}| = \sqrt{\tfrac{1}{2}(m_{11}+m_{12}+m_{21}+m_{22})}$$

$$|S_{AB}| = \sqrt{\tfrac{1}{2}(m_{11}-m_{12}+m_{21}-m_{22})}$$

$$|S_{BA}| = \sqrt{\tfrac{1}{2}(m_{11}+m_{12}-m_{21}-m_{22})}$$

$$|S_{BB}| = \sqrt{\tfrac{1}{2}(m_{11}-m_{12}-m_{21}+m_{22})}$$

$$\phi_{AA} = \phi_{AB} + \arctan\left(\frac{m_{14}+m_{24}}{m_{13}+m_{23}}\right)$$

ϕ_{AB} is arbitrary (may be = 0)

$$\phi_{BA} = \phi_{AB} + \arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right) + \arctan\left(\frac{m_{41}-m_{42}}{m_{31}-m_{32}}\right)$$

$$\phi_{BB} = \phi_{AB} + \arctan\left(\frac{m_{41}-m_{42}}{m_{31}-m_{32}}\right)$$

2. Monostatic case :

$$|S_{AA}| = \sqrt{\tfrac{1}{2}(m_{11}+2m_{12}+m_{22})}$$

$$|S_{AB}| = \sqrt{\tfrac{1}{2}(m_{11}-m_{22})} = |S_{BA}|$$

$$|S_{BB}| = \sqrt{\tfrac{1}{2}(m_{11}-2m_{12}+m_{22})}$$

$$\phi_{AA} = \phi_{AB}+\arctan\left(\frac{m_{14}+m_{24}}{m_{13}+m_{23}}\right)$$

ϕ_{AB} is arb.

$$\phi_{BA} = \phi_{AB}$$

$$\phi_{BB} = \phi_{AB}-\arctan\left(\frac{m_{14}-m_{24}}{m_{13}-m_{23}}\right)$$

Appendix H

Derivation of the Scattering Matrix[S] from the Modified Mueller Matrix[Mm] :

From Appendix F, Eq. (V), we have :

$$M_{11} = |S_{AA}|^2$$

$$M_{12} = |S_{AB}|^2$$

$$M_{13} = \mathrm{Re}\{S_{AA}S_{AB}^*\}$$

$$M_{14} = \mathrm{Im}\{S_{AA}S_{AB}^*\}$$

$$M_{21} = |S_{BA}|^2$$

$$M_{22} = |S_{BB}|^2$$

$$M_{23} = \mathrm{Re}\{S_{BA}S_{BB}^*\}$$

$$M_{24} = \mathrm{Im}\{S_{BA}S_{BB}^*\}$$

$$M_{31} = 2\mathrm{Re}\{S_{AA}S_{BA}^*\}$$

$$M_{32} = 2\mathrm{Re}\{S_{AB}S_{BB}^*\}$$

$$M_{33} = \mathrm{Re}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}$$

$$M_{34} = \mathrm{Im}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}$$

$$M_{41} = -2\mathrm{Im}\{S_{AA}S_{BA}^*\}$$

$$M_{42} = -2\mathrm{Im}\{S_{AB}S_{BB}^*\}$$

$$M_{43} = -\mathrm{Im}\{S_{AA}S_{BB}^*+S_{AB}S_{BA}^*\}$$

$$M_{44} = \mathrm{Re}\{S_{AA}S_{BB}^*-S_{AB}S_{BA}^*\}$$

Calculation of the amplititudes :

$$|S_{AA}| = \sqrt{M_{11}}$$

$$|S_{AB}| = \sqrt{M_{12}}$$

$$|S_{BA}| = \sqrt{M_{21}}$$

$$|S_{BB}| = \sqrt{M_{22}}$$

<u>Calculation of the phases</u> :

$$M_{13}+jM_{14} = \mathrm{Re}\{S_{AA}S_{AB}^{*}\}+j\mathrm{Im}\{S_{AA}S_{AB}^{*}\}$$

$$= S_{AA}S_{AB}^{*}$$

$$= |S_{AA}||S_{AB}|e^{j(\phi_{AA}-\phi_{AB})}$$

Therefore,

$$\tan(\phi_{AA}-\phi_{AB}) = \frac{M_{14}}{M_{13}}$$

Then,

$$\phi_{AA} = \phi_{AB}+\arctan\left(\frac{M_{14}}{M_{13}}\right)$$

also,

$$M_{34}+M_{43} = \mathrm{Im}\{S_{AA}S_{BB}^{*}-S_{AB}S_{BA}^{*}\}-\mathrm{Im}\{S_{AA}S_{BB}^{*}+S_{AB}S_{BA}^{*}\}$$

$$= \mathrm{Im}\{S_{AA}S_{BB}^{*}\}-\mathrm{Im}\{S_{AB}S_{BA}^{*}\}-\mathrm{Im}\{S_{AA}S_{BB}^{*}\}-\mathrm{Im}\{S_{AB}S_{BA}^{*}\}$$

$$= -2\mathrm{Im}\{S_{AB}S_{BA}^{*}\}$$

and

$$M_{33}-M_{44} = \mathrm{Re}\{S_{AA}S_{BB}^{*}+S_{AB}S_{BA}^{*}\}-\mathrm{Re}\{S_{AA}S_{BB}^{*}-S_{AB}S_{BA}^{*}\}$$

$$= 2\mathrm{Re}\{S_{AB}S_{BA}^{*}\}$$

then

$$(M_{33}-M_{44})-j(M_{34}+M_{43}) = 2[Fe\{S_{AB}S_{BA}^{*}\}+jIm\{S_{AB}S_{BA}^{*}\}$$

$$= 2S_{AB}S_{BA}^{*}$$

$$= 2|S_{AB}||S_{BA}|e^{j(\phi_{AB}-\phi_{BA})}$$

$$\tan(\phi_{AB}-\phi_{BA}) = -\left(\frac{M_{34}+M_{43}}{M_{33}-M_{44}}\right)$$

Therefore,

$$\phi_{AB}-\phi_{BA} = -\arctan\left(\frac{M_{34}+M_{43}}{M_{33}-M_{44}}\right)$$

or

$$\phi_{BA} = \phi_{AB}+\arctan\left(\frac{M_{34}+M_{43}}{M_{33}-M_{44}}\right)$$

also we have

$$M_{32}-jM_{42} = 2[Re\{S_{AB}S_{BB}^{*}\}+jIm\{S_{AB}S_{BB}^{*}\}]$$

$$= 2S_{AB}S_{BB}^{*}$$

$$= 2|S_{AB}||S_{BB}|e^{j(\phi_{AB}-\phi_{BB})}$$

then

$$\tan(\phi_{AB}-\phi_{BB}) = -\frac{M_{42}}{M_{32}}$$

or

$$\phi_{BB} = \phi_{AB}+\arctan\left(\frac{M_{42}}{M_{32}}\right)$$

In summary : for the bistatic case :

$$|S_{AA}| = \sqrt{M_{11}},\ |S_{AB}| = \sqrt{M_{12}},\ |S_{BA}| = \sqrt{M_{21}},\ |S_{BB}| = \sqrt{M_{22}}$$

and

$$\phi_{AA} = \phi_{AB} + \arctan\left(\frac{M_{14}}{M_{13}}\right)$$

ϕ_{AB} is arbitrary (may be=0)

$$\phi_{BA} = \phi_{AB} + \arctan\left(\frac{M_{34}+M_{43}}{M_{33}-M_{44}}\right)$$

$$\phi_{BB} = \phi_{AB} + \arctan\left(\frac{M_{42}}{M_{32}}\right)$$

simplify in the monostatic case so that $|S_{AB}|=|S_{BA}|$ and $\phi_{AB}=\phi_{BA}$.

APPENDIX (I)

COMPUTER PROGRAMS

This Appendix includes three computer programs. The first one calculates the Mueller matrices [M] and [Mm] from the Scattering matrix [S]. Also, it calculates the COPOL and XPOL nulls and their representation on the Poincare Sphere.

The second program reconstructs [S] with relative phase from [M] or [Mm].

The third one reconstructs also [S] from the knowledge of the spherical coordinates on the Poincare Sphere of either two COPOL nulls or one COPOL and one XPOL null .

I-1 FIRST COMPUTER PROGRAM :

This program calculates the Mueller matrices [M] and [Mm] f m the scattering matrix [S] in the bistatic and monostatic cases using the Equations of Section 3.6.1. Also, it calculates the COPOL and XPOL nulls and their representation on the Poincare sphere using the equations of Section 3.5.

```
//BAKRY  JOB
/*JOBPARM R=348
// EXEC WATFIV
//SYSIN DD *
$JOB NOEXT
C
C     THIS PROGRAM CALCULATES THE MUELLER AND THE MODIFIED MUELLER
C     MATRICES FROM THE SCATTERING MATRIX IN BISTATIC AND MONOSTATIC
C     CASES.
C     ALSO, IT CALCULATES THE COPOL AND XPOL NULLS FOR S AND REPRESENT
C     THEM ON POINCARE' SPHERE .
C     PUT LANE=0 IF SIGMA'S AND PHI'S ARE KNOWN
C     PUT LANE.NE.0 IF S(I,J) IS KNOWN
C
      COMPLEX S(2,2),RHO(2),CI
      DIMENSION AM(4,4),AMM(4,4),THETA(2),PHI(2)
      COMMON CI,PI,PI1,PI180
      CI=(0.0,1.0)
      PI=4.0*ATAN(1.0)
      PI1=180.0/PI
      PI180=PI/180
C
C     DATA
C
      NDATA=9
      LANE=0
      DO 999 II=1,NDATA
      IF(LANE.EQ.0) GO TO 160
      READ 55,((S(I,J),J=1,2),I=1,2)
   55 FORMAT(4E10.3)
      GO TO 161
  160 CONTINUE
C     SIGHH,SIGHV,SIGVV IN DECIBELS
C     PHIHH,PHIVV IN DEGREES
C
      READ(5,155)ASPECT,SIGHH,SIGHV,SIGVV,PHIHH,PHIVV
  155 FORMAT(6F10.2)
      WRITE(6,156)ASPECT,SIGHH,SIGHV,SIGVV,PHIHH,PHIVV
  156 FORMAT('1','ASPECT ANGLE = ',F10.2,2X,'DEGREES'//'0','SIGHH = '
     1,F10.2,2X,'DECIBELS'/'0','SIGHV = ',F10.2,2X,'DECIBELS'/'0',
     2'SIGVV = ',F10.2,2X,'DECIBELS'/'0','PHIHH = ',F10.2,2X,'DEGREES'
     3/'0','PHIVV = ',F10.2,2X,'DEGREES'//)
      CALL SCAT(SIGHH,SIGHV,SIGVV,PHIHH,PHIVV,S)
  161 CONTINUE
```

```
C     S(2,1)=S(1,2)
C
C     PWR=NORMALIZED FACTOR
C
C
      PWR=CABS(S(1,1))**2+CABS(S(2,2))**2+CABS(S(1,2))**2+CABS(S(2,1))**
     12
      PWR2=SQRT(PWR)
      WRITE(6,50)
   50 FORMAT('1','THE SCATTERING MATRIX IS :'//)
      DO 1 I=1,2
      WRITE(6,60)(S(I,J),J=1,2)
    1 CONTINUE
   60 FORMAT('0',2X,2(2E10.3,5X))
C
C     WRITE NORMALIZED SCATTERING MATRIX
C
      WRITE(6,51)
   51 FORMAT('0','THE NORMALIZED SCATTERING MATRIX IS :'//)
      DO 11 I=1,2
   11 WRITE(6,60)(S(I,J)/PWR2,J=1,2)
C
      CALL STOM(S,AM)
      WRITE(6,70)
   70 FORMAT('-','THE MUELLER MATRIX IS :'//)
      DO 2 I=1,4
      WRITE(6,80)(AM(I,J),J=1,4)
    2 CONTINUE
   80 FORMAT('0',2X,4(E10.3,5X))
      XX=AM(3,3)+AM(2,2)-AM(1,1)
      WRITE(6,*)XX,PWR,PWR2
C
C     WRITE NORMALIZED MUELLER MATRIX
C
      WRITE(6,71)
   71 FORMAT('0','THE NORMALIZED MUELLER MATRIX IS :'//)
      DO 12 I=1,4
   12 WRITE(6,80)(AM(I,J)/PWR,J=1,4)
      CALL STOMM(S,AMM)
      WRITE(6,90)
   90 FORMAT('1','THE MODIFIED MUELLER MATRIX IS :'//)
      DO 3 I=1,4
      WRITE(6,80)(AMM(I,J),J=1,4)
    3 CONTINUE
C
C     WRITE NORMALIZED MODIFIED MUELLER MATRIX
C
      WRITE(6,91)
   91 FORMAT('0','THE NORMALIZED MODIFIED MUELLER MATRIX IS :'//)
      DO 13 I=1,4
   13 WRITE(6,80)(AMM(I,J)/PWR,J=1,4)
C
      ISW=0
      CALL COPOL(S,RHO,THETA,PHI,ISW)
```

```
      IF(ISW.EQ.1) GO TO 120
      WRITE(6,100)
  100 FORMAT('1','THE COPOL NULLS ARE :'//)
      DO 4 I=1,2
      WRITE(6,101)RHO(I),THETA(I),PHI(I)
    4 CONTINUE
  101 FORMAT('0','RHO = ',2G14.7//'0','THETA = ',F8.3,2X,'DEGREES'//
     1'0','PHI = ',F8.3,2X,'DEGREES'//)
  120 CONTINUE
      CALL XPOL(S,RHO,THETA,PHI,ISW)
      IF(ISW.EQ.2) GO TO 999
      WRITE(6,110)
  110 FORMAT('0','THE XPOL NULLS ARE :'//)
      DO 5 I=1,2
      WRITE(6,101)RHO(I),THETA(I),PHI(I)
    5 CONTINUE
  999 CONTINUE
      STOP
      END
C
      SUBROUTINE STOM(S,AM)
C     THIS ROUTINE CALCULATES THE MUELLER MATRIX AM FROM SCATTERING
C     MATRIX S IN BISTATIC AND MONOSTATIC CASES.
C     G=(I,Q,U,V)
C
      COMPLEX S(2,2),CONJG,S1,S2
      DIMENSION AM(4,4)
      SAA=CABS(S(1,1))**2
      SAB=CABS(S(1,2))**2
      SBA=CABS(S(2,1))**2
      SBB=CABS(S(2,2))**2
      AM(1,1)=(SAA+SAB+SBA+SBB)/2.0
      AM(1,2)=(SAA-SAB+SBA-SBB)/2.0
      AM(2,1)=(SAA+SAB-SBA-SBB)/2.0
      AM(2,2)=(SAA-SAB-SBA+SBB)/2.0
      S1=S(1,1)*CONJG(S(1,2))
      S2=S(2,1)*CONJG(S(2,2))
      AM(1,3)=REAL(S1+S2)
      AM(1,4)=AIMAG(S1+S2)
      AM(2,3)=REAL(S1-S2)
      AM(2,4)=AIMAG(S1-S2)
      S1=S(1,1)*CONJG(S(2,1))
      S2=S(1,2)*CONJG(S(2,2))
      AM(3,1)=REAL(S1+S2)
      AM(3,2)=REAL(S1-S2)
      AM(4,1)=-AIMAG(S1+S2)
      AM(4,2)=-AIMAG(S1-S2)
      S1=S(1,1)*CONJG(S(2,2))
      S2=S(1,2)*CONJG(S(2,1))
      AM(3,3)=REAL(S1+S2)
      AM(3,4)=AIMAG(S1-S2)
      AM(4,3)=-AIMAG(S1+S2)
      AM(4,4)=REAL(S1-S2)
      RETURN
```

```
      END
C
      SUBROUTINE STOMM(S,AMM)
C     THIS ROUTINE CALCULATES THE MODIFIED MUELLER MATRIX AMM FROM THE
C     SCATTERING MATRIX S IN BISTATIC AND MONOSTATIC CASES.
C     GM=(IA,IB,U,V)
C
      COMPLEX S(2,2),CONJG,S1,S2
      DIMENSION AMM(4,4)
      AMM(1,1)=CABS(S(1,1))**2
      AMM(1,2)=CABS(S(1,2))**2
      S1=S(1,1)*CONJG(S(1,2))
      AMM(1,3)=REAL(S1)
      AMM(1,4)=AIMAG(S1)
      AMM(2,1)=CABS(S(2,1))**2
      AMM(2,2)=CABS(S(2,2))**2
      S1=S(2,1)*CONJG(S(2,2))
      AMM(2,3)=REAL(S1)
      AMM(2,4)=AIMAG(S1)
      S1=S(1,1)*CONJG(S(2,1))
      AMM(3,1)=2.0*REAL(S1)
      AMM(4,1)=-2.0*AIMAG(S1)
      S1=S(1,2)*CONJG(S(2,2))
      AMM(3,2)=2.0*REAL(S1)
      AMM(4,2)=-2.0*AIMAG(S1)
      S1=S(1,1)*CONJG(S(2,2))
      S2=S(1,2)*CONJG(S(2,1))
      AMM(3,3)=REAL(S1+S2)
      AMM(4,3)=-AIMAG(S1+S2)
      AMM(3,4)=AIMAG(S1-S2)
      AMM(4,4)=REAL(S1-S2)
      RETURN
      END
C
      SUBROUTINE COPOL(S,RHO,THETA,PHI,ISW)
C     THIS ROUTINE CALCULATES THE COPOL NULLS AND REPRESENT THEM ON THE
C     POINCARE' SPHERE .
C
      COMPLEX S(2,2),RHO(2),A,B,C,D,CSQRT
      DIMENSION THETA(2),PHI(2)
      A=S(2,2)
      B=S(1,2)+S(2,1)
      C=S(1,1)
      D=CSQRT(B*B-4.0*A*C)
      ERROR=1.0E-10
      WRITE(6,*)A,B,C,D
      ERRB=CABS(B)
      ERR=CABS(A)
      IF(ERR.LE.ERROR.AND.ERRB.LE.ERROR) GO TO 30
      IF(ERR.LE.ERROR) GO TO 10
      RHO(1)=(-B+D)/(2.0*A)
      RHO(2)=(-B-D)/(2.0*A)
      GO TO 20
   10 WRITE(6,50)ERR
```

```
   50 FORMAT('0','WE HAVE ONE ROOT BECAUSE A = ',G14.7/)
      RHO(1)=-C/B
      RHO(2)=RHO(1)
   20 CONTINUE
      CALL POINCR(RHO,THETA,PHI)
      GO TO 1
   30 CONTINUE
      ISW=1
      WRITE(6,40)
   40 FORMAT('0','A = B = 0.0 , NO ROOTS .....'/)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE XPOL(S,RHO,THETA,PHI,ISW)
C     THIS ROUTINE CALCULATES XPOL NULLS AND REPRESENT THEM ON THE
C     POINCARE' SPHERE :
C
      COMPLEX S(2,2),RHO(2),A,B,C,D,CONJG,CSQRT
      DIMENSION THETA(2),PHI(2)
      A=S(2,2)*CONJG(S(2,1))+S(2,1)*CONJG(S(1,1))
      B=S(2,2)*CONJG(S(2,2))-S(1,1)*CONJG(S(1,1))
      B=B-S(1,2)*CONJG(S(2,1))+S(2,1)*CONJG(S(1,2))
      C=S(1,1)*CONJG(S(1,2))+S(1,2)*CONJG(S(2,2))
      C=-C
      D=CSQRT(B*B-4.0*A*C)
      ERROR=1.0E-10
      ERRB=CABS(B)
      ERR=CABS(A)
      WRITE(6,*)A,B,C,D
      IF(ERR.LE.ERROR.AND.ERRB.LE.ERROR) GO TO 30
      IF(ERR.LE.ERROR) GO TO 10
      RHO(1)=(B+D)/(2.0*A)
      RHO(2)=(B-D)/(2.0*A)
      GO TO 20
   10 WRITE(6,50)ERR
   50 FORMAT('0','WE HAVE ONE ROOT BECAUSE A = ',G14.7/)
      RHO(1)=C/B
      RHO(2)=RHO(1)
   20 CONTINUE
      CALL POINCR(RHO,THETA,PHI)
      GO TO 1
   30 ISW=2
      WRITE(6,40)
   40 FORMAT('0','A = B = 0.0 ,  NO ROOTS ........'/)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE POINCR(RHO,THETA,PHI)
      COMPLEX RHO(2),CI,Q,CONJG,YY
      DIMENSION THETA(2),PHI(2)
      COMMON CI,PI,PI1,PI180
      AMIN=1.E-10
```

```
      DO 1 I=1,2
      YY=1.0+CI*RHO(I)
      YY1=CABS(YY)
      IF(YY1.LE.AMIN) GO TO 10
      Q=(1.0-CI*RHO(I))/YY
      Q2=CABS(Q)**2
      Q3=-(1.0-Q2)/(1.0+Q2)
      W1=AIMAG(Q)
      W2=REAL(Q)
      IF(ABS(W1).LE.AMIN.AND.ABS(W2).LE.AMIN) GO TO 40
      PHI(I)=-ATAN2(W1,W2)
      PHI(I)=PHI(I)*PI1
      GO TO 20
   40 WRITE(6,50)
   50 FORMAT('0','PHI IS ARBITRARY , REAL AND IMAG Q ARE ZERO'/)
      GO TO 20
   10 CONTINUE
      Q3=1.0
      WRITE(6,30)
   30 FORMAT('0','Q IS INFINITY , PHI IS ARBITRARY '/)
   20 THETA(I)=ARCOS(Q3)
      THETA(I)=THETA(I)*PI1
    1 CONTINUE
      RETURN
      END
      SUBROUTINE SCAT(SIGHH,SIGHV,SIGVV,PHIHH,PHIVV,S)
      COMPLEX CI,S(2,2)
      COMMON CI,PI,PI1,PI180
      PRINT,'CI , PI = ',CI,PI
      PHIHH=PHIHH*PI180
      PHIVV=PHIVV*PI180
      SIGHH=SIGHH/20.0
      S1=10.0**SIGHH
      S(1,1)=S1*(COS(PHIHH)+CI*SIN(PHIHH))
      SIGHV=SIGHV/20.0
      S2=10.0**SIGHV
      S(1,2)=S2
      S(2,1)=S(1,2)
      SIGVV=SIGVV/20.0
      S3=10.0**SIGVV
      S(2,2)=S3*(COS(PHIVV)+CI*SIN(PHIVV))
      PRINT,'MAG(S(1,1)),MAG(S(1,2)),MAG(S(2,2)) = ',S1,S2,S3
      RETURN
      END
$ENTRY
      0.0       3.1     -24.4       4.6     203.5     193.5
      9.50      0.5     -14.9       2.1    -170.2    -156.2
     26.0       3.6     -16.1       2.8     128.0     121.0
     42.75      1.3     -29.5       1.2      65.1      75.6
     77.71     21.5     -18.8      21.5      -8.3     -10.8
     96.0      -0.5     -22.1      -0.8     -36.0     -45.5
    108.0     -23.2     -24.8      -5.1     108.3    -162.2
    125.50     -1.4     -28.5       0.6     -22.9     -60.4
    172.0      17.3     -15.2      18.4    -133.4    -131.4
```

I-2 SECOND COMPUTER PROGRAM :

This program reconstrucs the scattering matrix [S] with relative phase from [M] or [Mm] using the equations of Section 3.6.2.

```
//BAKRY  JOB
/*JOBPARM R=348
// EXEC WATFIV
//SYSIN DD *
$JOB NOEXT
C
C     THIS PROGRAM CALCULATES THE RELATIVE PHASE SCATTERING MATRIX (S)
C     FROM MUELLER (M) AND/OR MODIFIED MUELLER (MM) MATRICES IN BISTATIC
C     AND MONOSTATIC CASES.
C
C     DATA : MDATA = NUMBER OF (M)'S MATRICES TO BE ENTERED AS DATA .
C            MMDATA =    ,,    ,,(MM)'S    ,,     ,, ,,    ,,    ,,  ,,  .
C     MON=0 FOR MONOSTATIC , MON <> 0 FOR BISTATIC
C     NROW= NUMBER OF ROWS
C
      COMPLEX S(2,2),CI
      DIMENSION AM(4,4),AS(2,2),PHI(2,2)
      COMMON PI,PI1,CI,MON
      PI=4.0*ATAN(1.0)
      PI1=180.0/PI
      CI=(0.0,1.0)
C     WRITE(6,*)PI,CI
      MON=0
      NROW=4
      IF(MON.EQ.0) NROW=2
      MDATA = 3
      MMDATA = 3
      NDATA = MDATA + MMDATA
      DO 999 II=1,NDATA
      READ 55,((AM(I,J),J=1,4),I=1,NROW)
   55 FORMAT(4E10.3)
      IF(II.GT.MDATA) GO TO 10
      WRITE(6,50)
   50 FORMAT('1','THE INPUT DATA OF THE MUELLER MATRIX IS :'//)
      DO 1 I=1,NROW
    1 WRITE(6,60)(AM(I,J),J=1,4)
   60 FORMAT('0',2X,4(E10.3,5X))
      CALL AMTOS(AM,AS,PHI)
      GO TO 20
   10 WRITE(6,70)
   70 FORMAT('1','THE INPUT DATA OF THE MODIFIED MUELLER MATRIX IS :'//)
      DO 2 I=1,NROW
    2 WRITE(6,60)(AM(I,J),J=1,4)
      CALL AMMTOS(AM,AS,PHI)
   20 CONTINUE
      DO 30 J=1,2
      DO 30 I=1,2
      S(I,J)=AS(I,J)*(COS(PHI(I,J))+CI*SIN(PHI(I,J)))
```

```
      PHI(I,J)=PHI(I,J)*PI1
   30 CONTINUE
      WRITE(6,80)PHI(1,2)
   80 FORMAT('-','THE SCATTERING MATRIX WITH ARBITRARY PHASE PHIAB = ',
     1F8.3,2X,'DEGREES'//)
      DO 3 I=1,2
    3 WRITE(6,60)(S(I,J),J=1,2)
      WRITE(6,90)
   90 FORMAT('-','THE SCATTERING MATRIX IN POLAR FORM IS :'//)
      DO 4 I=1,2
    4 WRITE(6,60)(AS(I,J),PHI(I,J),J=1,2)
  999 CONTINUE
      STOP
      END
C
      SUBROUTINE AMTOS(AM,AS,PHI)
C
C     THIS ROUTINE CALCULATES THE RELATIVE PHASE SCATTERING MATRIX FROM
C     MUELLER MATRIX IN BISTATIC AND MONOSTATIC CASES .
C     PHI(1,2) IS ARBITRARY E.G. ( = 0.0 )
C
      COMPLEX CI
      DIMENSION AM(4,4),AS(2,2),PHI(2,2)
      COMMON PI,PI1,CI,MON
C     WRITE(6,*)PI,CI
C
C     CALCULATE THE MAGNITUDES OF (S)
C
      IF(MON.NE.0) GO TO 10
      AM(2,1)=AM(1,2)
      AM(3,1)=AM(1,3)
      AM(3,2)=AM(2,3)
      AM(4,1)=-AM(1,4)
      AM(4,2)=-AM(2,4)
   10 CONTINUE
      AS(1,1)=SQRT((AM(1,1)+AM(1,2)+AM(2,1)+AM(2,2))/2.0)
      AS(1,2)=SQRT((AM(1,1)-AM(1,2)+AM(2,1)-AM(2,2))/2.0)
      AS(2,1)=SQRT((AM(1,1)+AM(1,2)-AM(2,1)-AM(2,2))/2.0)
      AS(2,2)=SQRT((AM(1,1)-AM(1,2)-AM(2,1)+AM(2,2))/2.0)
C
C     CALCULATE THE RELATIVE PHASES OF (S)
C
      PHI(1,2)=0.0
      X=AM(1,3)+AM(2,3)
      Y=AM(1,4)+AM(2,4)
      PHI(1,1)=PHI(1,2)+ATAN2(Y,X)
      X=AM(3,1)-AM(3,2)
      Y=AM(4,1)-AM(4,2)
      PHI(2,2)=PHI(1,2)+ATAN2(Y,X)
      X=AM(1,3)-AM(2,3)
      Y=AM(1,4)-AM(2,4)
      PHI(2,1)=PHI(2,2)+ATAN2(Y,X)
C     PRINT,'PKI(2,1) = ',PHI(2,1)
      IF(MON.EQ.0) PHI(2,1)=PHI(1,2)
```

```
      RETURN
      END
C
      SUBROUTINE AMMTOS(AM,AS,PHI)
C
C     THIS ROUTINE CALCULATES THE RELATIVE PHASE SCATTERING MATRIX FROM
C     THE MODIFIED MUELLER MATRIX IN BISTATIC AND MONOSTATIC CASES.
C     PHI(1,2) IS ARBITRARY E.G. ( = 0.0 )
C
      COMPLEX CI
      DIMENSION AM(4,4),AS(2,2),PHI(2,2)
      COMMON PI,PI1,CI,MON
C     WRITE(6,*)PI,CI
C
C     CALCULATES THE MAGNITUDES OF (S)
C
      IF(MON.NE.0) GO TO 10
      AM(2,1)=AM(1,2)
      AM(3,2)=2.0*AM(2,3)
      AM(4,2)=-2.0*AM(2,4)
   10 CONTINUE
      DO 1 I=1,2
      DO 1 J=1,2
      AS(I,J)=SQRT(AM(I,J))
    1 CONTINUE
C
C     CALCULATES THE RELATIVE PHASES
C
      PHI(1,2)=0.0
      PHI(1,1)=PHI(1,2)+ATAN2(AM(1,4),AM(1,3))
      PHI(2,2)=PHI(1,2)+ATAN2(AM(4,2),AM(3,2))
      IF(MON.EQ.0) GO TO 20
      Y=AM(3,4)+AM(4,3)
      X=AM(3,3)-AM(4,4)
      PHI(2,1)=PHI(1,2)+ATAN2(Y,X)
      GO TO 21
   20 PHI(2,1)=PHI(1,2)
   21 CONTINUE
      RETURN
      END
$ENTRY
 0.500E 00-0.494E 00 0.679E-01-0.179E-01
           0.491E 00-0.631E-01 0.206E-01
 0.500E 00-0.403E 00 0.251E 00-0.884E-01
           0.368E 00-0.169E 00 0.124E 00
 0.500E 00 0.454E-01-0.606E-01 0.350E-03
           0.488E 00-0.815E-02 0.877E-01
 0.162E-02 0.471E-02 0.239E-02 0.138E-02
           0.989E 00 0.655E-01-0.193E-01
 0.306E-01 0.662E-01 0.413E-01 0.178E-01
           0.837E 00 0.210E 00-0.106E 00
 0.540E 00 0.578E-02-0.344E-01 0.440E-01
           0.449E 00-0.262E-01-0.437E-01
```

I-3 THIRD COMPUTER PROGRAM :

This program reconstructs the scattering matrix [S] from the knowledge of the spherical coordinates on the Poincare sphere of two COPOL nulls or one COPOL and one XPOL null using the equations of Section 3.7.

```
//BAKRY  JOB
/*JOBPARM R=348
//GO EXEC WATFIV,PARM=NOEXT
//SYSIN DD *
$JOB
C
C     THIS PROGRAM RECONSTRUCTS THE SCATTERING MATRIX (S) WITH RELATIVE
C     PHASE GIVEN ITS COPOL NULLS (P,THETA,PHI), WHERE P,THETA AND PHI
C     ARE THE RADIUS, COLATITUDE AND LONGITUDE OF THE COPOL NULLS ON THE
C     POINCARE' SPHERE OR ONE COPOL AND ONE XPOL NULL.
C     DATA: PUT NCC (TWO COPOL NULLS) FIRST.
C           PUT NCX (ONE COPOL AND ONE XPOL NULL) SECOND.
C
      DIMENSION THETA(2),PHI(2)
      COMPLEX CI,Q,RHO(2),R1,R2,AK,S(2,2),S1,Q1,R3,CONJG
      CI=(0.0,1.0)
      PI=4.0*ATAN(1.0)
      PI180=PI/180.0
      PI1=180.0/PI
C     PRINT,PI,CI
      AMIN=1.E-03
      AMAX=1.E+10
      NCC=12
      NCX=5
      NDATA=NCC+NCX
      DO 10 II=1,NDATA
      IF(II.LE.NCC) WRITE(6,100)
      IF(II.GT.NCC) WRITE(6,110)
      READ(5,50)P,(THETA(I),PHI(I),I=1,2)
      WRITE(6,60)P,(THETA(I),PHI(I),I=1,2)
      WRITE(6,70)
      DO 1 I=1,2
      THR=THETA(I)*PI180
      PHR=PHI(I)*PI180
      X=1.0+COS(THR)
      Y=1.0-COS(THR)
      IF(Y.LE.1.E-07) GO TO 20
      Z=SQRT(X/Y)
      GO TO 21
   20 Z=AMAX
   21 CONTINUE
      Q=Z*(COS(PHR)-CI*SIN(PHR))
      Q1=1.0+Q
C     PRINT,'Q1 = ',Q1
      IF(CABS(Q1).LE.1.E-03) GO TO 30
C     IF(CABS(Q1).LE.AMIN) Q1=1.0/AMAX
```

```
      RHO(I)=-CI*(1.0-Q)/Q1
      GO TO 31
   30 RHO(I)=-CI*AMAX
   31 CONTINUE
      WRITE(6,71)THETA(I),PHI(I),Q,RHO(I)
    1 CONTINUE
      IF(II.GT.NCC) GO TO 40
C     PRINT,'**RHO(1),RHO(2) = ',RHO(1),RHO(2)
      R1=RHO(1)+RHO(2)
      R2=RHO(1)*RHO(2)
      R1MAG=CABS(R1)
      X1=AIMAG(R1)
      X2=REAL(R1)
      IF(ABS(X1).LE.AMIN.AND.ABS(X2).LE.AMIN) GO TO 45
      R1PHAS=ATAN2(X1,X2)
      GO TO 46
   45 CONTINUE
      R1PHAS=0.0
   46 CONTINUE
C     PRINT,'**TEST**',R1,R2,X1,X2,R1PHAS
      R2MAG=CABS(R2)
      X=R1MAG**2+2.0*R2MAG**2+2.0
      X=2.0*X
      AK=SQRT(P/X)
      S1=COS(R1PHAS)-CI*SIN(R1PHAS)
      S(1,1)=-2.0*R2*S1
      S(1,2)=R1MAG
      S(2,1)=S(1,2)
      S(2,2)=-2.0*S1
      GO TO 41
   40 CONTINUE
C
C     RHO(1)............. COPOL NULL
C     RHO(2)............. XPOL NULL
C
      R1=RHO(1)**2*CONJG(RHO(2))+RHO(2)
      R1MAG=CABS(R1)
      R1PHAS=ATAN2(AIMAG(R1),REAL(R1))
      R2=RHO(1)-RHO(1)*CABS(RHO(2))**2-2.0*RHO(2)
      R3=2.0*RHO(1)*CONJG(RHO(2))-CABS(RHO(2))**2+1.0
      R2MAG=CABS(R2)
      R3MAG=CABS(R3)
      D=2.0*R1MAG**2+(CABS(RHO(1))*R2MAG)**2+R3MAG**2
      AK=SQRT(P/D)
      S1=COS(R1PHAS)-CI*SIN(R1PHAS)
      S(1,1)=RHO(1)*S1*R2
      S(1,2)=R1MAG
      S(2,1)=S(1,2)
      S(2,2)=-R3*S1
   41 CONTINUE
      WRITE(6,80)AK
      DO 2 I=1,2
      WRITE(6,81)(S(I,J),J=1,2)
    2 CONTINUE
```

```
      WRITE(6,90)
      DO 3 I=1,2
      WRITE(6,81)(S(I,J)*AK,J=1,2)
    3 CONTINUE
   10 CONTINUE
      STOP
   50 FORMAT(5F10.3)
   60 FORMAT('0','THE INPUT DATA IS :'//'0','P = ',F10.3/'0',
     1'THETA(1) = ',F10.3,2X,'DEGREES',10X,'PHI(1) = ',F10.3,2X,
     2'DEGREES'/'0','THETA(2) = ',F10.3,2X,'DEGREES',10X,'PHI(2) = ',
     3F10.3,2X,'DEGREES'//)
   70 FORMAT('0',2X,'THETA(DEG.)',5X,'PHI(DEG.)',11X,'Q',19X,'RHO'/)
   71 FORMAT('0',2(2X,F10.3),2(2X,2E10.3))
   80 FORMAT('1','THE RECONSTRUCTED SCATTERING MATRIX WITHOUT MULTIPLYIN
     1G BY THE CONSTANT K = ',2F10.3,2X,'IS : '//)
   81 FORMAT('0',2(2X,2F10.3))
   90 FORMAT('0','THE SCATTERING MATRIX IS : '//)
  100 FORMAT('1','THE COPOL NULLS ARE KNOWN'//)
  110 FORMAT('1','ONE COPOL NULL AND ONE XPOL NULL ARE KNOWN'//)
      END
$ENTRY
 1.0      109.414   -4.055    66.486     -11.708
 1.0      118.001   -22.209   41.311     -45.001
 1.0      172.57     70.438    7.736     179.957
 1.0      139.467   -31.889   40.877      58.378
 1.0      177.921   -28.335    0.934     -31.12
 2.0        0.0               180.0
 2.0       90.0     -90.0     90.0        90.0
 1.0      180.0               180.0
 1.0        0.0                 0.0
 1.0       90.0     180.0     90.0       180.0
 1.0       90.0       0.0     90.0         0.0
 1.0       90.0     270.0     90.0       270.0
 1.0      109.414    -4.055   92.054     172.172
 1.0      118.001   -22.209   100.543     148.062
 1.0      172.57     70.438   89.735     -53.177
 1.0      139.467   -31.889   89.756    -166.555
 1.0      177.921   -28.335   90.568     150.796
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